PREFACE

This *Complete Solutions Manual* contains solutions to all of the exercises in my textbook *Applied Calculus for the Managerial, Life, and Social Sciences: A Brief Approach, Tenth Edition*. The corresponding *Student Solutions Manual* contains solutions to the odd-numbered exercises and the even-numbered exercises in the "Before Moving On" quizzes. It also offers problem-solving tips for many sections.

I would like to thank Andy Bulman-Fleming for checking the accuracy of the answers to the new exercises in this edition of the text, rendering the art, and typesetting this manual. I also wish to thank my development editor Laura Wheel and my editor Rita Lombard of Cengage Learning for their help and support in bringing this supplement to market.

Please submit any errors in the solutions manual or suggestions for improvements to me in care of the publisher: Math Editorial, Cengage Learning, 20 Channel Center Street, Boston, MA, 02210.

Soo T. Tan

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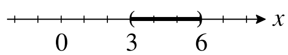
1 PRELIMINARIES

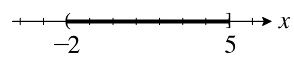
1.1 Precalculus Review I

Exercises page 13

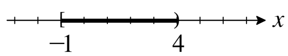
1. The interval $(3, 6)$ is shown on the number line below. Note that this is an open interval indicated by "(" and ")".



2. The interval $(-2, 5]$ is shown on the number line below.

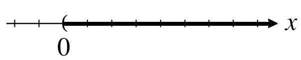


3. The interval $[-1, 4)$ is shown on the number line below. Note that this is a half-open interval indicated by "**[**'' (closed) and "**)**"(open).

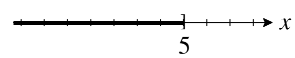


4. The closed interval $\left[-\frac{6}{5}, -\frac{1}{2}\right]$ is shown on the number line below.

5. The infinite interval $(0, \infty)$ is shown on the number line below.



6. The infinite interval $(-\infty, 5]$ is shown on the number line below.



7. $27^{2/3} = \left(3^3\right)^{2/3} = 3^2 = 9.$

8. $8^{-4/3} = \left(\dfrac{1}{8^{4/3}}\right) = \dfrac{1}{2^4} = \dfrac{1}{16}.$

9. $\left(\frac{1}{\sqrt{3}}\right)^0 = 1.$ Recall that any number raised to the zeroth power is 1.

10. $\left(7^{1/2}\right)^4 = 7^{4/2} = 7^2 = 49.$

11. $\left[\left(\frac{1}{8}\right)^{1/3}\right]^{-2} = \left(\frac{1}{2}\right)^{-2} = \left(2^2\right) = 4.$

12. $\left[\left(-\frac{1}{3}\right)^2\right]^{-3} = \left(\frac{1}{9}\right)^{-3} = (9)^3 = 729.$

13. $\left(\dfrac{7^{-5} \cdot 7^2}{7^{-2}}\right)^{-1} = \left(7^{-5+2+2}\right)^{-1} = \left(7^{-1}\right)^{-1} = 7^1 = 7.$

14. $\left(\frac{9}{16}\right)^{-1/2} = \left(\frac{16}{9}\right)^{1/2} = \frac{4}{3}.$

15. $\left(125^{2/3}\right)^{-1/2} = 125^{(2/3)(-1/2)} = 125^{-1/3} = \dfrac{1}{125^{1/3}}$
$= \frac{1}{5}.$

16. $\sqrt[3]{2^6} = \left(2^6\right)^{1/3} = 2^{6(1/3)} = 2^2 = 4.$

17. $\dfrac{\sqrt{32}}{\sqrt{8}} = \sqrt{\dfrac{32}{8}} = \sqrt{4} = 2.$

18. $\sqrt[3]{\dfrac{-8}{27}} = \dfrac{\sqrt[3]{-8}}{\sqrt[3]{27}} = -\dfrac{2}{3}.$

19. $\dfrac{16^{5/8}16^{1/2}}{16^{7/8}} = 16^{(5/8)+(1/2)-(7/8)} = 16^{1/4} = 2.$

20. $\left(\dfrac{9^{-3}\cdot 9^{5}}{9^{-2}}\right)^{-1/2} = 9^{(-3+5+2)(-1/2)} = 9^{4(-1/2)} = \dfrac{1}{81}.$

21. $16^{1/4}\cdot 8^{-1/3} = 2\cdot\left(\frac{1}{8}\right)^{1/3} = 2\cdot\frac{1}{2} = 1.$

22. $\dfrac{6^{2.5}\cdot 6^{-1.9}}{6^{-1.4}} = 6^{2.5-1.9-(-1.4)} = 6^{2.5-1.9+1.4} = 6^{2}$
$= 36.$

23. True.

24. True. $3^2 \times 2^2 = (3\times 2)^2 = 6^2 = 36.$

25. False. $x^3 \times 2x^2 = 2x^{3+2} = 2x^5 \neq 2x^6.$

26. False. $3^3 + 3 = 27 + 3 = 30 \neq 3^4.$

27. False. $\dfrac{2^{4x}}{1^{3x}} = \dfrac{2^{4x}}{1} = 2^{4x}.$

28. True. $\left(2^2\times 3^2\right)^2 = (4\times 9)^2 = 36^2 = \left(6^2\right)^2 = 6^4.$

29. False. $\dfrac{1}{4^{-3}} = 4^3 = 64.$

30. True. $\dfrac{4^{3/2}}{2^4} = \dfrac{8}{16} = \dfrac{1}{2}.$

31. False. $\left(1.2^{1/2}\right)^{-1/2} = (1.2)^{-1/4} \neq 1.$

32. True.
$5^{2/3}\times 25^{2/3} = 5^{2/3}\left(5^2\right)^{2/3} = 5^{2/3}\times 5^{4/3} = 5^2 = 25.$

33. $(xy)^{-2} = \dfrac{1}{(xy)^2}.$

34. $3s^{1/3}\cdot s^{-7/3} = 3s^{(1/3)-(7/3)} = 3s^{-6/3} = 3s^{-2} = \dfrac{3}{s^2}.$

35. $\dfrac{x^{-1/3}}{x^{1/2}} = x^{(-1/3)-(1/2)} = x^{-5/6} = \dfrac{1}{x^{5/6}}.$

36. $\sqrt{x^{-1}}\cdot\sqrt{9x^{-3}} = x^{-1/2}\cdot 3x^{-3/2} = 3x^{(-1/2)+(-3/2)}$
$= 3x^{-2} = \dfrac{3}{x^2}.$

37. $12^0\,(s+t)^{-3} = 1\cdot\dfrac{1}{(s+t)^3} = \dfrac{1}{(s+t)^3}.$

38. $(x-y)\left(x^{-1}+y^{-1}\right) = (x-y)\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$
$= (x-y)\left(\dfrac{y+x}{xy}\right) = \dfrac{(x-y)(x+y)}{xy} = \dfrac{x^2-y^2}{xy}.$

39. $\dfrac{x^{7/3}}{x^{-2}} = x^{(7/3)+2} = x^{(7/3)+(6/3)} = x^{13/3}.$

40. $\left(49x^{-2}\right)^{-1/2} = (49)^{-1/2}\,x^{(-2)(-1/2)} = \frac{1}{7}x.$

41. $\left(x^2y^{-3}\right)\left(x^{-5}y^3\right) = x^{2-5}y^{-3+3} = x^{-3}y^0 = x^{-3} = \dfrac{1}{x^3}.$

42. $\dfrac{5x^6y^3}{2x^2y^7} = \frac{5}{2}x^{6-2}y^{3-7} = \frac{5}{2}x^4y^{-4} = \dfrac{5x^4}{2y^4}.$

43. $\dfrac{x^{3/4}}{x^{-1/4}} = x^{(3/4)-(-1/4)} = x^{4/4} = x.$

44. $\left(\dfrac{x^3y^2}{z^2}\right)^2 = \dfrac{x^{3\cdot 2}y^{2\cdot 2}}{z^{2(2)}} = \dfrac{x^6y^4}{z^4}.$

45. $\left(\dfrac{x^3}{-27y^{-6}}\right)^{-2/3} = x^{3(-2/3)}\left(-\dfrac{1}{27}\right)^{-2/3}y^{6(-2/3)}$
$= x^{-2}\left(-\dfrac{1}{3}\right)^{-2}y^{-4} = \dfrac{9}{x^2y^4}.$

46. $\left(\dfrac{e^x}{e^{x-2}}\right)^{-1/2} = e^{[x-(x-2)](-1/2)} = e^{-1} = \dfrac{1}{e}.$

47. $\left(\dfrac{x^{-3}}{y^{-2}}\right)^2\left(\dfrac{y}{x}\right)^4 = \dfrac{x^{-3\cdot 2}y^4}{y^{-2\cdot 2}x^4} = \dfrac{y^{4+4}}{x^{4+6}} = \dfrac{y^8}{x^{10}}.$

48. $\dfrac{(r^n)^4}{r^{5-2n}} = r^{4n-(5-2n)} = r^{4n+2n-5} = r^{6n-5}.$

49. $\sqrt[3]{x^{-2}} \cdot \sqrt{4x^5} = x^{-2/3} \cdot 4^{1/2} \cdot x^{5/2} = x^{(-2/3)+(5/2)} \cdot 2 = 2x^{11/6}.$

50. $\sqrt{81x^6y^{-4}} = (81)^{1/2} \cdot x^{6/2} \cdot y^{-4/2} = \dfrac{9x^3}{y^2}.$

51. $-\sqrt[4]{16x^4y^8} = -\left(16^{1/4} \cdot x^{4/4} \cdot y^{8/4}\right) = -2xy^2.$

52. $\sqrt[3]{x^{3a+b}} = x^{(3a+b)(1/3)} = x^{a+(b/3)}.$

53. $\sqrt[6]{64x^8y^3} = 64^{1/6} \cdot x^{8/6}y^{3/6} = 2x^{4/3}y^{1/2}.$

54. $\sqrt[3]{27r^6} \cdot \sqrt{s^2t^4} = 27^{1/3}\left(r^6\right)^{1/3}\left(s^2\right)^{1/2}\left(t^4\right)^{1/2} = 3r^2st^2.$

55. $2^{3/2} = 2\left(2^{1/2}\right) \approx 2\,(1.414) = 2.828.$

56. $8^{1/2} = \left(2^3\right)^{1/2} = 2^{3/2} = 2\left(2^{1/2}\right) \approx 2.828.$

57. $9^{3/4} = \left(3^2\right)^{3/4} = 3^{6/4} = 3^{3/2} = 3 \cdot 3^{1/2} \approx 3\,(1.732) = 5.196.$

58. $6^{1/2} = (2 \cdot 3)^{1/2} = 2^{1/2} \cdot 3^{1/2} \approx (1.414)\,(1.732) \approx 2.449.$

59. $10^{3/2} = 10^{1/2} \cdot 10 \approx (3.162)\,(10) = 31.62.$

60. $1000^{3/2} = \left(10^3\right)^{3/2} = 10^{9/2} = 10^4 \times 10^{1/2} \approx (10000)\,(3.162) = 31{,}620.$

61. $10^{2.5} = 10^2 \cdot 10^{1/2} \approx 100\,(3.162) = 316.2.$

62. $(0.0001)^{-1/3} = \left(10^{-4}\right)^{-1/3} = 10^{4/3} = 10 \cdot 10^{1/3} \approx 10\,(2.154) = 21.54.$

63. $\dfrac{3}{2\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{3\sqrt{x}}{2x}.$

64. $\dfrac{3}{\sqrt{xy}} \cdot \dfrac{\sqrt{xy}}{\sqrt{xy}} = \dfrac{3\sqrt{xy}}{xy}.$

65. $\dfrac{2y}{\sqrt{3y}} \cdot \dfrac{\sqrt{3y}}{\sqrt{3y}} = \dfrac{2y\sqrt{3y}}{3y} = \dfrac{2\sqrt{3y}}{3}.$

66. $\dfrac{5x^2}{\sqrt{3x}} \cdot \dfrac{\sqrt{3x}}{\sqrt{3x}} = \dfrac{5x^2\sqrt{3x}}{3x} = \dfrac{5x\sqrt{3x}}{3}.$

67. $\dfrac{1}{\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^3}} = \dfrac{\sqrt[3]{x^2}}{x}.$

68. $\sqrt{\dfrac{2x}{y}} = \dfrac{\sqrt{2x}}{\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{\sqrt{2xy}}{y}.$

69. $\dfrac{2\sqrt{x}}{3} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{2x}{3\sqrt{x}}.$

70. $\dfrac{\sqrt[3]{x}}{24} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{x}{24\sqrt[3]{x^2}}.$

71. $\sqrt{\dfrac{2y}{x}} = \dfrac{\sqrt{2y}}{\sqrt{x}} \cdot \dfrac{\sqrt{2y}}{\sqrt{2y}} = \dfrac{2y}{\sqrt{2xy}}.$

72. $\sqrt[3]{\dfrac{2x}{3y}} = \dfrac{\sqrt[3]{2x}}{\sqrt[3]{3y}} \cdot \dfrac{\sqrt[3]{(2x)^2}}{\sqrt[3]{(2x)^2}} = \dfrac{2x}{\sqrt[3]{12x^2y}}.$

73. $\dfrac{\sqrt[3]{x^2z}}{y} \cdot \dfrac{\sqrt[3]{xz^2}}{\sqrt[3]{xz^2}} = \dfrac{\sqrt[3]{x^3z^3}}{y\sqrt[3]{xz^2}} = \dfrac{xz}{y\sqrt[3]{xz^2}}.$

74. $\dfrac{\sqrt[3]{x^2y}}{2x} \cdot \dfrac{\sqrt[3]{xy^2}}{\sqrt[3]{xy^2}} = \dfrac{xy}{2x\sqrt[3]{xy^2}} = \dfrac{y}{2\sqrt[3]{xy^2}}.$

75. $\left(7x^2 - 2x + 5\right) + \left(2x^2 + 5x - 4\right) = 7x^2 - 2x + 5 + 2x^2 + 5x - 4 = 9x^2 + 3x + 1.$

76. $\left(3x^2 + 5xy + 2y\right) + \left(4 - 3xy - 2x^2\right) = 3x^2 + 5xy + 2y + 4 - 3xy - 2x^2 = x^2 + 2xy + 2y + 4.$

77. $\left(5y^2 - 2y + 1\right) - \left(y^2 - 3y - 7\right) = 5y^2 - 2y + 1 - y^2 + 3y + 7 = 4y^2 + y + 8.$

78. $3\,(2a - b) - 4\,(b - 2a) = 6a - 3b - 4b + 8a = 14a - 7b = 7\,(2a - b).$

79. $x - \{2x - [-x - (1-x)]\} = x - \{2x - [-x - 1 + x]\} = x - (2x+1) = x - 2x - 1 = -x - 1.$

80. $3x^2 - \left\{x^2 + 1 - x\,[x - (2x-1)]\right\} + 2 = 3x^2 - \left[x^2 + 1 - x\,(x - 2x + 1)\right] + 2$

$$= 3x^2 - \left[x^2 + 1 - x\,(-x+1)\right] + 2 = 3x^2 - \left(x^2 + 1 + x^2 - x\right) + 2$$

$$= 3x^2 - \left(2x^2 - x + 1\right) + 2 = x^2 - 1 + x + 2 = x^2 + x + 1.$$

81. $\left(\dfrac{1}{3} - 1 + e\right) - \left(-\dfrac{1}{3} - 1 + e^{-1}\right) = \dfrac{1}{3} - 1 + e + \dfrac{1}{3} + 1 - \dfrac{1}{e} = \dfrac{2}{3} + e - \dfrac{1}{e} = \dfrac{3e^2 + 2e - 3}{3e}.$

82. $-\frac{3}{4}y - \frac{1}{4}x + 100 + \frac{1}{2}x + \frac{1}{4}y - 120 = -\frac{3}{4}y + \frac{1}{4}y - \frac{1}{4}x + \frac{1}{2}x + 100 - 120 = -\frac{1}{2}y + \frac{1}{4}x - 20.$

83. $3\sqrt{8} + 8 - 2\sqrt{y} + \frac{1}{2}\sqrt{x} - \frac{3}{4}\sqrt{y} = 3\sqrt{8} + 8 + \frac{1}{2}\sqrt{x} - \frac{11}{4}\sqrt{y} = 6\sqrt{2} + 8 + \frac{1}{2}\sqrt{x} - \frac{11}{4}\sqrt{y}.$

84. $\frac{8}{9}x^2 + \frac{2}{3}x + \frac{16}{3}x^2 - \frac{16}{3}x - 2x + 2 = \dfrac{8x^2 + 6x + 48x^2 - 48x - 18x + 18}{9} = \dfrac{56x^2 - 60x + 18}{9}$

$$= \tfrac{2}{9}\left(28x^2 - 30x + 9\right).$$

85. $(x+8)(x-2) = x(x-2) + 8(x-2) = x^2 - 2x + 8x - 16 = x^2 + 6x - 16.$

86. $(5x+2)(3x-4) = 5x(3x-4) + 2(3x-4) = 15x^2 - 20x + 6x - 8 = 15x^2 - 14x - 8.$

87. $(a+5)^2 = (a+5)(a+5) = a(a+5) + 5(a+5) = a^2 + 5a + 5a + 25 = a^2 + 10a + 25.$

88. $(3a-4b)^2 = (3a-4b)(3a-4b) = 3a(3a-4b) - 4b(3a-4b) = 9a^2 - 12ab - 12ab + 16b^2$

$$= 9a^2 - 24ab + 16b^2.$$

89. $(x+2y)^2 = (x+2y)(x+2y) = x(x+2y) + 2y(x+2y) = x^2 + 2xy + 2yx + 4y^2 = x^2 + 4xy + 4y^2.$

90. $(6-3x)^2 = (6-3x)(6-3x) = 6(6-3x) - 3x(6-3x) = 36 - 18x - 18x + 9x^2 = 36 - 36x + 9x^2.$

91. $(2x+y)(2x-y) = 2x(2x-y) + y(2x-y) = 4x^2 - 2xy + 2xy - y^2 = 4x^2 - y^2.$

92. $(3x+2)(2-3x) = 3x(2-3x) + 2(2-3x) = 6x - 9x^2 + 4 - 6x = -9x^2 + 4.$

93. $\left(2x^2-1\right)\left(3x^2\right) + \left(x^2+3\right)(4x) = 6x^4 - 3x^2 + 4x^3 + 12x = 6x^4 + 4x^3 - 3x^2 + 12x = x\left(6x^3 + 4x^2 - 3x + 12\right).$

94. $\left(x^2-1\right)(2x) - x^2(2x) = 2x^3 - 2x - 2x^3 = -2x.$

95. $6x\left(\frac{1}{2}\right)\left(2x^2+3\right)^{-1/2}(4x) + 6\left(2x^2+3\right)^{1/2} = 3\left(2x^2+3\right)^{-1/2}\left[x(4x) + 2\left(2x^2+3\right)\right] = \dfrac{6\left(4x^2+3\right)}{\left(2x^2+3\right)^{1/2}}.$

96. $\left(x^{1/2}+1\right)\left(\frac{1}{2}x^{-1/2}\right) - \left(x^{1/2}-1\right)\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}\left[\left(x^{1/2}+1\right) - \left(x^{1/2}-1\right)\right] = \frac{1}{2}x^{-1/2}(2) = \dfrac{1}{x^{1/2}} = \dfrac{1}{\sqrt{x}}.$

97. $100\left(-10te^{-0.1t} - 100e^{-0.1t}\right) = -1000(10+t)e^{-0.1t}.$

98. $2\left(t+\sqrt{t}\right)^2 - 2t^2 = 2\left(t+\sqrt{t}\right)\left(t+\sqrt{t}\right) - 2t^2 = 2\left(t^2 + 2t\sqrt{t} + t\right) - 2t^2 = 2t^2 + 4t\sqrt{t} + 2t - 2t^2$

$$= 4t\sqrt{t} + 2t = 2t\left(2\sqrt{t} + 1\right).$$

99. $4x^5 - 12x^4 - 6x^3 = 2x^3\left(2x^2 - 6x - 3\right)$.

100. $4x^2y^2z - 2x^5y^2 + 6x^3y^2z^2 = 2x^2y^2\left(2z - x^3 + 3xz^2\right)$.

101. $7a^4 - 42a^2b^2 + 49a^3b = 7a^2\left(a^2 + 7ab - 6b^2\right)$.

102. $3x^{2/3} - 2x^{1/3} = x^{1/3}\left(3x^{1/3} - 2\right)$.

103. $e^{-x} - xe^{-x} = e^{-x}(1 - x)$.

104. $2ye^{xy^2} + 2xy^3e^{xy^2} = 2ye^{xy^2}\left(1 + xy^2\right)$.

105. $2x^{-5/2} - \frac{3}{2}x^{-3/2} = \frac{1}{2}x^{-5/2}(4 - 3x)$.

106. $\frac{1}{2}\left(\frac{2}{3}u^{3/2} - 2u^{1/2}\right) = \frac{1}{2}\cdot\frac{2}{3}u^{1/2}(u - 3) = \frac{1}{3}u^{1/2}(u - 3)$.

107. $6ac + 3bc - 4ad - 2bd = 3c(2a + b) - 2d(2a + b) = (2a + b)(3c - 2d)$.

108. $3x^3 - x^2 + 3x - 1 = x^2(3x - 1) + 1(3x - 1) = \left(x^2 + 1\right)(3x - 1)$.

109. $4a^2 - b^2 = (2a + b)(2a - b)$, a difference of two squares.

110. $12x^2 - 3y^2 = 3\left(4x^2 - y^2\right) = 3(2x + y)(2x - y)$.

111. $10 - 14x - 12x^2 = -2\left(6x^2 + 7x - 5\right) = -2(3x + 5)(2x - 1)$.

112. $x^2 - 2x - 15 = (x - 5)(x + 3)$.

113. $3x^2 - 6x - 24 = 3\left(x^2 - 2x - 8\right) = 3(x - 4)(x + 2)$.

114. $3x^2 - 4x - 4 = (3x + 2)(x - 2)$.

115. $12x^2 - 2x - 30 = 2\left(6x^2 - x - 15\right) = 2(3x - 5)(2x + 3)$.

116. $(x + y)^2 - 1 = (x + y - 1)(x + y + 1)$.

117. $9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x - 4y)(3x + 4y)$.

118. $8a^2 - 2ab - 6b^2 = 2\left(4a^2 - ab - 3b^2\right) = 2(a - b)(4a + 3b)$.

119. $x^6 + 125 = \left(x^2\right)^3 + (5)^3 = \left(x^2 + 5\right)\left(x^4 - 5x^2 + 25\right)$.

120. $x^3 - 27 = x^3 - 3^3 = (x - 3)\left(x^2 + 3x + 9\right)$.

121. $\left(x^2 + y^2\right)x - xy(2y) = x^3 + xy^2 - 2xy^2 = x^3 - xy^2$.

122. $2kr(R - r) - kr^2 = 2kRr - 2kr^2 - kr^2 = 2kRr - 3kr^2 = kr(2R - 3r)$.

123. $2(x-1)(2x+2)^3[4(x-1)+(2x+2)] = 2(x-1)(2x+2)^3(4x-4+2x+2)$
$$= 2(x-1)(2x+2)^3(6x-2) = 4(x-1)(3x-1)(2x+2)^3$$
$$= 32(x-1)(3x-1)(x+1)^3.$$

124. $5x^2\left(3x^2+1\right)^4(6x)+\left(3x^2+1\right)^5(2x) = (2x)\left(3x^2+1\right)^4\left[15x^2+\left(3x^2+1\right)\right] = 2x\left(3x^2+1\right)^4\left(18x^2+1\right)$.

125. $4(x-1)^2(2x+2)^3(2)+(2x+2)^4(2)(x-1) = 2(x-1)(2x+2)^3[4(x-1)+(2x+2)]$
$$= 2(x-1)(2x+2)^3(6x-2) = 4(x-1)(3x-1)(2x+2)^3$$
$$= 32(x-1)(3x-1)(x+1)^3.$$

126. $\left(x^2+1\right)\left(4x^3-3x^2+2x\right)-\left(x^4-x^3+x^2\right)(2x) = 4x^5-3x^4+2x^3+4x^3-3x^2+2x-2x^5+2x^4-2x^3$
$$= 2x^5-x^4+4x^3-3x^2+2x.$$

127. $\left(x^2+2\right)^2\left[5\left(x^2+2\right)^2-3\right](2x) = \left(x^2+2\right)^2\left[5\left(x^4+4x^2+4\right)-3\right](2x) = (2x)\left(x^2+2\right)^2\left(5x^4+20x^2+17\right)$.

128. $\left(x^2-4\right)\left(x^2+4\right)(2x+8)-\left(x^2+8x-4\right)\left(4x^3\right) = \left(x^4-16\right)(2x+8)-4x^5-32x^4+16x^3$
$$= 2x^5+8x^4-32x-128-4x^5-32x^4+16x^3 = -2x^5-24x^4+16x^3-32x-128$$
$$= -2\left(x^5+12x^4-8x^3+16x+64\right).$$

129. We factor the left-hand side of $x^2+x-12=0$ to obtain $(x+4)(x-3)=0$, so $x=-4$ or $x=3$. We conclude that the roots are $x=-4$ and $x=3$.

130. We factor the left-hand side of $3x^2-x-4=0$ to obtain $(3x-4)(x+1)=0$. Thus, $3x=4$ or $x=-1$, and we conclude that the roots are $x=\frac{4}{3}$ and $x=-1$.

131. $4t^2+2t-2=(2t-1)(2t+2)=0$. Thus, the roots are $t=\frac{1}{2}$ and $t=-1$.

132. $-6x^2+x+12=(3x+4)(-2x+3)=0$. Thus, $x=-\frac{4}{3}$ and $x=\frac{3}{2}$ are the roots of the equation.

133. $\frac{1}{4}x^2-x+1=\left(\frac{1}{2}x-1\right)\left(\frac{1}{2}x-1\right)=0$. Thus $\frac{1}{2}x=1$, and so $x=2$ is a double root of the equation.

134. $\frac{1}{2}a^2+a-12=a^2+2a-24=(a+6)(a-4)=0$. Thus, $a=-6$ and $a=4$ are the roots of the equation.

135. We use the quadratic formula to solve the equation $4x^2+5x-6=0$. In this case, $a=4$, $b=5$, and $c=-6$. Therefore, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-5\pm\sqrt{5^2-4(4)(-6)}}{2(4)}=\dfrac{-5\pm\sqrt{121}}{8}=\dfrac{-5\pm 11}{8}$. Thus, $x=-\frac{16}{8}=-2$ and $x=\frac{6}{8}=\frac{3}{4}$ are the roots of the equation.

136. We use the quadratic formula to solve the equation $3x^2-4x+1=0$. Here $a=3$, $b=-4$, and $c=1$, so $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(-4)\pm\sqrt{(-4)^2-4(3)(1)}}{2(3)}=\dfrac{4\pm\sqrt{4}}{6}$. Thus, $x=\frac{6}{6}=1$ and $x=\frac{2}{6}=\frac{1}{3}$ are the roots of the equation.

137. We use the quadratic formula to solve the equation $8x^2 - 8x - 3 = 0$. Here $a = 8$, $b = -8$, and $c = -3$, so $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(8)(-3)}}{2(8)} = \dfrac{8 \pm \sqrt{160}}{16} = \dfrac{8 \pm 4\sqrt{10}}{16} = \dfrac{2 \pm \sqrt{10}}{4}$. Thus, $x = \frac{1}{2} + \frac{1}{4}\sqrt{10}$ and $x = \frac{1}{2} - \frac{1}{4}\sqrt{10}$ are the roots of the equation.

138. We use the quadratic formula to solve the equation $x^2 - 6x + 6 = 0$. Here $a = 1$, $b = -6$, and $c = 6$. Therefore, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(6)}}{2(1)} = \dfrac{6 \pm 2\sqrt{3}}{2} = 3 \pm \sqrt{3}$. Thus, the roots are $3 + \sqrt{3}$ and $3 - \sqrt{3}$.

139. We use the quadratic formula to solve $2x^2 + 4x - 3 = 0$. Here $a = 2$, $b = 4$, and $c = -3$, so $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)} = \dfrac{-4 \pm \sqrt{40}}{4} = \dfrac{-4 \pm 2\sqrt{10}}{4} = \dfrac{-2 \pm \sqrt{10}}{2}$. Thus, $x = -1 + \frac{1}{2}\sqrt{10}$ and $x = -1 - \frac{1}{2}\sqrt{10}$ are the roots of the equation.

140. We use the quadratic formula to solve the equation $2x^2 + 7x - 15 = 0$. Then $a = 2$, $b = 7$, and $c = -15$. Therefore, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)} = \dfrac{-7 \pm \sqrt{169}}{4} = \dfrac{-7 \pm 13}{4}$. We conclude that $x = \frac{3}{2}$ and $x = -5$ are the roots of the equation.

141. The total revenue is given by $\left(0.2t^2 + 150t\right) + \left(0.5t^2 + 200t\right) = 0.7t^2 + 350t$ thousand dollars t months from now, where $0 \le t \le 12$.

142. In month t, the revenue of the second gas station will exceed that of the first gas station by $\left(0.5t^2 + 200t\right) - \left(0.2t^2 + 150t\right) = 0.3t^2 + 50t$ thousand dollars, where $0 < t \le 12$.

143. **a.** $f(30{,}000) = \left(5.6 \times 10^{11}\right)(30{,}000)^{-1.5} \approx 107{,}772$, or 107,772 families.

b. $f(60{,}000) = \left(5.6 \times 10^{11}\right)(60{,}000)^{-1.5} \approx 38{,}103$, or 38,103 families.

c. $f(150{,}000) = \left(5.6 \times 10^{11}\right)(150{,}000)^{-1.5} \approx 9639$, or 9639 families.

144. $-t^3 + 6t^2 + 15t = -t\left(t^2 - 6t - 15\right)$.

145. $8000x - 100x^2 = 100x(80 - x)$.

146. True. The two real roots are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

147. True. If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is not a real number.

148. True, because $(a + b)(b - a) = b^2 - a^2$.

1.2 Precalculus Review II

Exercises page 23

1. $$\frac{x^2+x-2}{x^2-4}=\frac{(x+2)(x-1)}{(x+2)(x-2)}=\frac{x-1}{x-2}.$$

2. $$\frac{2a^2-3ab-9b^2}{2ab^2+3b^3}=\frac{(2a+3b)(a-3b)}{b^2(2a+3b)}=\frac{a-3b}{b^2}.$$

3. $$\frac{12t^2+12t+3}{4t^2-1}=\frac{3\left(4t^2+4t+1\right)}{4t^2-1}=\frac{3(2t+1)(2t+1)}{(2t+1)(2t-1)}=\frac{3(2t+1)}{2t-1}.$$

4. $$\frac{x^3+2x^2-3x}{-2x^2-x+3}=\frac{x\left(x^2+2x-3\right)}{-\left(2x^2+x-3\right)}=\frac{x(x+3)(x-1)}{-(2x+3)(x-1)}=-\frac{x(x+3)}{2x+3}.$$

5. $$\frac{(4x-1)(3)-(3x+1)(4)}{(4x-1)^2}=\frac{12x-3-12x-4}{(4x-1)^2}=-\frac{7}{(4x-1)^2}.$$

6. $$\frac{\left(1+x^2\right)^2(2)-2x(2)\left(1+x^2\right)(2x)}{\left(1+x^2\right)^4}=\frac{\left(1+x^2\right)(2)\left(1+x^2-4x^2\right)}{\left(1+x^2\right)^4}=\frac{\left(1+x^2\right)(2)\left(-3x^2+1\right)}{\left(1+x^2\right)^4}=\frac{2\left(1-3x^2\right)}{\left(1+x^2\right)^3}.$$

7. $$\frac{2a^2-2b^2}{b-a}\cdot\frac{4a+4b}{a^2+2ab+b^2}=\frac{2(a+b)(a-b)4(a+b)}{-(a-b)(a+b)(a+b)}=-8.$$

8. $$\frac{x^2-6x+9}{x^2-x-6}\cdot\frac{3x+6}{2x^2-7x+3}=\frac{3(x-3)^2(x+2)}{(x-3)(x+2)(2x-1)(x-3)}=\frac{3}{2x-1}.$$

9. $$\frac{3x^2+2x-1}{2x+6}\div\frac{x^2-1}{x^2+2x-3}=\frac{(3x-1)(x+1)}{2(x+3)}\cdot\frac{(x+3)(x-1)}{(x+1)(x-1)}=\frac{3x-1}{2}.$$

10. $$\frac{3x^2-4xy-4y^2}{x^2y}\div\frac{(2y-x)^2}{x^3y}=\frac{(3x+2y)(x-2y)}{x^2y}\cdot\frac{x^3y}{(2y-x)(2y-x)}=\frac{x(3x+2y)}{x-2y}.$$

11. $$\frac{58}{3(3t+2)}+\frac{1}{3}=\frac{58+3t+2}{3(3t+2)}=\frac{3t+60}{3(3t+2)}=\frac{t+20}{3t+2}.$$

12. $$\frac{a+1}{3a}+\frac{b-2}{5b}=\frac{5b(a+1)+3a(b-2)}{15ab}=\frac{5ab+5b+3ab-6a}{15ab}=\frac{-6a+8ab+5b}{15ab}.$$

13. $$\frac{2x}{2x-1}-\frac{3x}{2x+5}=\frac{2x(2x+5)-3x(2x-1)}{(2x-1)(2x+5)}=\frac{4x^2+10x-6x^2+3x}{(2x-1)(2x+5)}=\frac{-2x^2+13x}{(2x-1)(2x+5)}$$
$$=-\frac{x(2x-13)}{(2x-1)(2x+5)}.$$

14. $$\frac{-xe^x}{x+1}+e^x=\frac{-xe^x+(x+1)e^x}{x+1}=\frac{-xe^x+xe^x+e^x}{x+1}=\frac{e^x}{x+1}.$$

15. $$\frac{4}{x^2-9}-\frac{5}{x^2-6x+9}=\frac{4}{(x+3)(x-3)}-\frac{5}{(x-3)^2}=\frac{4(x-3)-5(x+3)}{(x-3)^2(x+3)}=-\frac{x+27}{(x-3)^2(x+3)}.$$

16. $\dfrac{x}{1-x}+\dfrac{2x+3}{x^2-1}=\dfrac{-x\,(x+1)+2x+3}{(x+1)\,(x-1)}=\dfrac{-x^2-x+2x+3}{x^2-1}=-\dfrac{x^2-x-3}{x^2-1}.$

17. $\dfrac{1+\dfrac{1}{x}}{1-\dfrac{1}{x}}=\dfrac{\dfrac{x+1}{x}}{\dfrac{x-1}{x}}=\dfrac{x+1}{x}\cdot\dfrac{x}{x-1}=\dfrac{x+1}{x-1}.$

18. $\dfrac{\dfrac{1}{x}+\dfrac{1}{y}}{1-\dfrac{1}{xy}}=\dfrac{\dfrac{x+y}{xy}}{\dfrac{xy-1}{xy}}=\dfrac{x+y}{xy}\cdot\dfrac{xy}{xy-1}=\dfrac{x+y}{xy-1}.$

19. $\dfrac{4x^2}{2\sqrt{2x^2+7}}+\sqrt{2x^2+7}=\dfrac{4x^2+2\sqrt{2x^2+7}\sqrt{2x^2+7}}{2\sqrt{2x^2+7}}=\dfrac{4x^2+4x^2+14}{2\sqrt{2x^2+7}}=\dfrac{4x^2+7}{\sqrt{2x^2+7}}.$

20. $$6\,(2x+1)^2\sqrt{x^2+x}+\frac{(2x+1)^4}{2\sqrt{x^2+x}}=\frac{6\,(2x+1)^2\sqrt{x^2+x}\,(2)\sqrt{x^2+x}+(2x+1)^4}{2\sqrt{x^2+x}}$$
$$=\frac{(2x+1)^2\left[12\left(x^2+x\right)+4x^2+4x+1\right]}{2\sqrt{x^2+x}}=\frac{(2x+1)^2\left(16x^2+16x+1\right)}{2\sqrt{x^2+x}}.$$

21. $5\left[\dfrac{\left(t^2+1\right)(1)-t\,(2t)}{\left(t^2+1\right)^2}\right]=\dfrac{5\left(t^2+1-2t^2\right)}{\left(t^2+1\right)^2}=\dfrac{5\left(1-t^2\right)}{\left(t^2+1\right)^2}=-\dfrac{5\left(t^2-1\right)}{\left(t^2+1\right)^2}.$

22. $\dfrac{2x\,(x+1)^{-1/2}-(x+1)^{1/2}}{x^2}=\dfrac{(x+1)^{-1/2}\,(2x-x-1)}{x^2}=\dfrac{(x+1)^{-1/2}\,(x-1)}{x^2}=\dfrac{x-1}{x^2\sqrt{x+1}}.$

23. $\dfrac{\left(x^2+1\right)^2(-2)+(2x)\,2\left(x^2+1\right)(2x)}{\left(x^2+1\right)^4}=\dfrac{2\left(x^2+1\right)\left[-\left(x^2+1\right)+4x^2\right]}{\left(x^2+1\right)^4}=\dfrac{2\left(3x^2-1\right)}{\left(x^2+1\right)^3}.$

24. $\dfrac{\left(x^2+1\right)^{1/2}-2x^2\left(x^2+1\right)^{-1/2}}{1-x^2}=\dfrac{\left(x^2+1\right)^{-1/2}\left(x^2+1-2x^2\right)}{1-x^2}=\dfrac{\left(x^2+1\right)^{-1/2}\left(-x^2+1\right)}{1-x^2}=\dfrac{1}{\sqrt{x^2+1}}.$

25. $3\left(\dfrac{2x+1}{3x+2}\right)^2\left[\dfrac{(3x+2)\,(2)-(2x+1)\,(3)}{(3x+2)^2}\right]=\dfrac{3\,(2x+1)^2\,(6x+4-6x-3)}{(3x+2)^4}=\dfrac{3\,(2x+1)^2}{(3x+2)^4}.$

26. $\dfrac{(2x+1)^{1/2}-(x+2)\,(2x+1)^{-1/2}}{2x+1}=\dfrac{(2x+1)^{-1/2}\,(2x+1-x-2)}{2x+1}=\dfrac{(2x+1)^{-1/2}\,(x-1)}{2x+1}=\dfrac{x-1}{(2x+1)^{3/2}}.$

27. $$100\left[\frac{\left(t^2+20t+100\right)(2t+10)-\left(t^2+10t+100\right)(2t+20)}{\left(t^2+20t+100\right)^2}\right]$$
$$=100\left[\frac{2t^3+40t^2+200t+10t^2+200t+1000-2t^3-20t^2-200t-20t^2-200t-2000}{\left(t^2+20t+100\right)^2}\right]$$
$$=\frac{100\left(10t^2-1000\right)}{\left(t^2+20t+100\right)^2}=\frac{1000\,(t-10)}{(t+10)^3}.$$

28. $$\frac{2(2x-3)^{1/3}-(x-1)(2x-3)^{-2/3}}{(2x-3)^{2/3}}=\frac{(2x-3)^{-2/3}[2(2x-3)-(x-1)]}{(2x-3)^{2/3}}=\frac{(2x-3)^{-2/3}(4x-6-x+1)}{(2x-3)^{2/3}}=\frac{3x-5}{(2x-3)^{4/3}}.$$

29. $\frac{1}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{\sqrt{3}+1}{3-1}=\frac{\sqrt{3}+1}{2}.$

30. $\frac{1}{\sqrt{x}+5}\cdot\frac{\sqrt{x}-5}{\sqrt{x}-5}=\frac{\sqrt{x}-5}{x-25}.$

31. $\frac{1}{\sqrt{x}-\sqrt{y}}\cdot\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\frac{\sqrt{x}+\sqrt{y}}{x-y}.$

32. $\frac{a}{1-\sqrt{a}}\cdot\frac{1+\sqrt{a}}{1+\sqrt{a}}=\frac{a\left(1+\sqrt{a}\right)}{1-a}.$

33. $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{a-b}.$

34. $\frac{2\sqrt{a}+\sqrt{b}}{2\sqrt{a}-\sqrt{b}}\cdot\frac{2\sqrt{a}+\sqrt{b}}{2\sqrt{a}+\sqrt{b}}=\frac{\left(2\sqrt{a}+\sqrt{b}\right)^2}{4a-b}.$

35. $\frac{\sqrt{x}}{3}\cdot\frac{\sqrt{x}}{\sqrt{x}}=\frac{x}{3\sqrt{x}}.$

36. $\frac{\sqrt[3]{y}}{x}\cdot\frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}}=\frac{y}{x\sqrt[3]{y^2}}.$

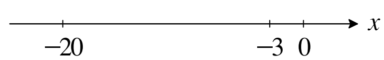
37. $\frac{1-\sqrt{3}}{3}\cdot\frac{1+\sqrt{3}}{1+\sqrt{3}}=\frac{1^2-\left(\sqrt{3}\right)^2}{3\left(1+\sqrt{3}\right)}=-\frac{2}{3\left(1+\sqrt{3}\right)}.$

38. $\frac{\sqrt{x}-1}{x}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}=\frac{x-1}{x\left(\sqrt{x}+1\right)}.$

39. $\frac{1+\sqrt{x+2}}{\sqrt{x+2}}\cdot\frac{1-\sqrt{x+2}}{1-\sqrt{x+2}}=\frac{1-(x+2)}{\sqrt{x+2}\left(1-\sqrt{x+2}\right)}=-\frac{x+1}{\sqrt{x+2}\left(1-\sqrt{x+2}\right)}.$

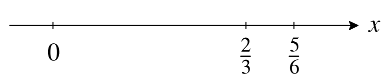
40. $\frac{\sqrt{x+3}-\sqrt{x}}{3}\cdot\frac{\sqrt{x+3}+\sqrt{x}}{\sqrt{x+3}+\sqrt{x}}=\frac{x+3-x}{3\left(\sqrt{x+3}+\sqrt{x}\right)}=\frac{1}{\sqrt{x+3}+\sqrt{x}}.$

41. The statement is false because -3 is greater than -20. See the number line below.



42. The statement is true because -5 is equal to -5.

43. The statement is false because $\frac{2}{3}=\frac{4}{6}$ is less than $\frac{5}{6}$.



44. The statement is false because $-\frac{5}{6}=-\frac{10}{12}$ is greater than $-\frac{11}{12}$.

45. We are given $2x+4<8$. Add -4 to each side of the inequality to obtain $2x<4$, then multiply each side of the inequality by $\frac{1}{2}$ to obtain $x<2$. We write this in interval notation as $(-\infty,2)$.

46. We are given $-6>4+5x$. Add -4 to each side of the inequality to obtain $-6-4>5x$, so $-10>5x$. Dividing by 2, we obtain $-2>x$, so $x<-2$. We write this in interval notation as $(-\infty,-2)$.

47. We are given the inequality $-4x\geq 20$. Multiply both sides of the inequality by $-\frac{1}{4}$ and reverse the sign of the inequality to obtain $x\leq -5$. We write this in interval notation as $(-\infty,-5]$.

48. $-12 \leq -3x \Rightarrow 4 \geq x$, or $x \leq 4$. We write this in interval notation as $(-\infty, 4]$.

49. We are given the inequality $-6 < x - 2 < 4$. First add 2 to each member of the inequality to obtain $-6 + 2 < x < 4 + 2$ and $-4 < x < 6$, so the solution set is the open interval $(-4, 6)$.

50. We add -1 to each member of the given double inequality $0 \leq x + 1 \leq 4$ to obtain $-1 \leq x \leq 3$, and the solution set is $[-1, 3]$.

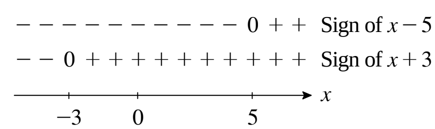
51. We want to find the values of x that satisfy at least one of the inequalities $x + 1 > 4$ and $x + 2 < -1$. Adding -1 to both sides of the first inequality, we obtain $x + 1 - 1 > 4 - 1$, so $x > 3$. Similarly, adding -2 to both sides of the second inequality, we obtain $x + 2 - 2 < -1 - 2$, so $x < -3$. Therefore, the solution set is $(-\infty, -3) \cup (3, \infty)$.

52. We want to find the values of x that satisfy at least one of the inequalities $x + 1 > 2$ and $x - 1 < -2$. Solving these inequalities, we find that $x > 1$ or $x < -1$, and the solution set is $(-\infty, -1) \cup (1, \infty)$.

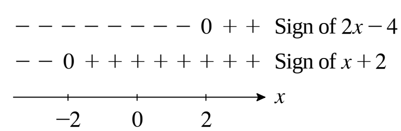
53. We want to find the values of x that satisfy the inequalities $x + 3 > 1$ and $x - 2 < 1$. Adding -3 to both sides of the first inequality, we obtain $x + 3 - 3 > 1 - 3$, or $x > -2$. Similarly, adding 2 to each side of the second inequality, we obtain $x - 2 + 2 < 1 + 2$, so $x < 3$. Because both inequalities must be satisfied, the solution set is $(-2, 3)$.

54. We want to find the values of x that satisfy the inequalities $x - 4 \leq 1$ and $x + 3 > 2$. Solving these inequalities, we find that $x \leq 5$ and $x > -1$, and the solution set is $(-1, 5]$.

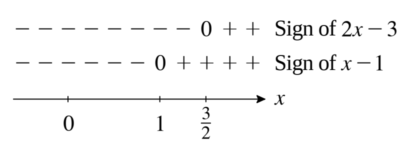
55. We want to find the values of x that satisfy the inequality $(x + 3)(x - 5) \leq 0$. From the sign diagram, we see that the given inequality is satisfied when $-3 \leq x \leq 5$, that is, when the signs of the two factors are different or when one of the factors is equal to zero.



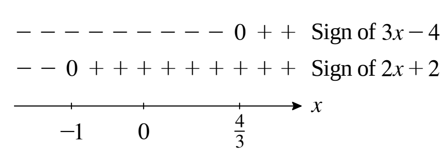
56. We want to find the values of x that satisfy the inequality $(2x - 4)(x + 2) \geq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq -2$ or $x \geq 2$; that is, when the signs of both factors are the same or one of the factors is equal to zero.



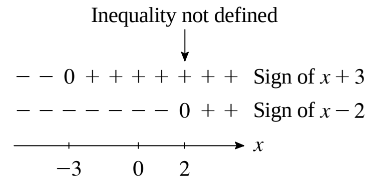
57. We want to find the values of x that satisfy the inequality $(2x - 3)(x - 1) \geq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq 1$ or $x \geq \frac{3}{2}$; that is,when the signs of both factors are the same, or one of the factors is equal to zero.



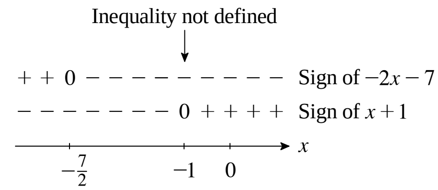
58. We want to find the values of x that satisfy the inequalities $(3x - 4)(2x + 2) \leq 0$.
From the sign diagram, we see that the given inequality is satisfied when $-1 \leq x \leq \frac{4}{3}$, that is, when the signs of the two factors are different or when one of the factors is equal to zero.



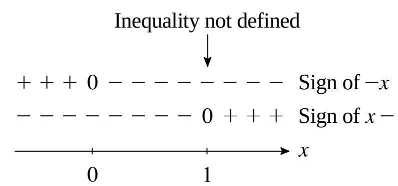
59. We want to find the values of x that satisfy the inequalities $\dfrac{x+3}{x-2} \geq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq -3$ or $x > 2$, that is, when the signs of the two factors are the same. Notice that $x = 2$ is not included because the inequality is not defined at that value of x.



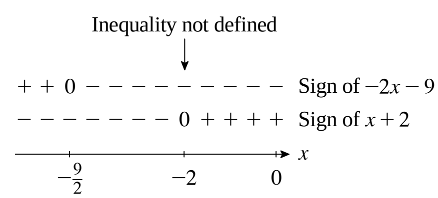
60. We want to find the values of x that satisfy the inequality $\dfrac{2x-3}{x+1} \geq 4$. We rewrite the inequality as $\dfrac{2x-3}{x+1} - 4 \geq 0$, $\dfrac{2x-3-4x-4}{x+1} \geq 0$, and $\dfrac{-2x-7}{x+1} \geq 0$. From the sign diagram, we see that the given inequality is satisfied when $-\frac{7}{2} \leq x < -1$; that is, when the signs of the two factors are the same. Notice that $x = -1$ is not included because the inequality is not defined at that value of x.



61. We want to find the values of x that satisfy the inequality $\dfrac{x-2}{x-1} \leq 2$. Subtracting 2 from each side of the given inequality and simplifying gives $\dfrac{x-2}{x-1} - 2 \leq 0$, $\dfrac{x-2-2(x-1)}{x-1} \leq 0$, and $-\dfrac{x}{x-1} \leq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq 0$ or $x > 1$; that is, when the signs of the two factors differ. Notice that $x = 1$ is not included because the inequality is undefined at that value of x.



62. We want to find the values of x that satisfy the inequality $\dfrac{2x-1}{x+2} \leq 4$. Subtracting 4 from each side of the given inequality and simplifying gives $\dfrac{2x-1}{x+2} - 4 \leq 0$, $\dfrac{2x-1-4(x+2)}{x+2} \leq 0$, $\dfrac{2x-1-4x-8}{x+2} \leq 0$, and finally $\dfrac{-2x-9}{x+2} \leq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq -\frac{9}{2}$ or $x > -2$.



63. $|-6+2| = 4$.

64. $4 + |-4| = 4 + 4 = 8$.

65. $\dfrac{|-12+4|}{|16-12|} = \dfrac{|-8|}{|4|} = 2$.

66. $\left|\dfrac{0.2-1.4}{1.6-2.4}\right| = \left|\dfrac{-1.2}{-0.8}\right| = 1.5$.

67. $\sqrt{3}\,|-2| + 3\left|-\sqrt{3}\right| = \sqrt{3}\,(2) + 3\sqrt{3} = 5\sqrt{3}$.

68. $|-1| + \sqrt{2}\,|-2| = 1 + 2\sqrt{2}$.

69. $|\pi - 1| + 2 = \pi - 1 + 2 = \pi + 1$.

70. $|\pi - 6| - 3 = 6 - \pi - 3 = 3 - \pi$.

71. $\left|\sqrt{2}-1\right| + \left|3-\sqrt{2}\right| = \sqrt{2} - 1 + 3 - \sqrt{2} = 2$.

72. $\left|2\sqrt{3}-3\right| - \left|\sqrt{3}-4\right| = 2\sqrt{3} - 3 - \left(4-\sqrt{3}\right) = 3\sqrt{3} - 7.$

73. False. If $a > b$, then $-a < -b$, $-a + b < -b + b$, and $b - a < 0$.

74. False. Let $a = -2$ and $b = -3$. Then $a/b = \frac{-2}{-3} = \frac{2}{3} < 1$.

75. False. Let $a = -2$ and $b = -3$. Then $a^2 = 4$ and $b^2 = 9$, and $4 < 9$. Note that we need only to provide a counterexample to show that the statement is not always true.

76. False. Let $a = -2$ and $b = -3$. Then $\dfrac{1}{a} = -\dfrac{1}{2}$ and $\dfrac{1}{b} = -\dfrac{1}{3}$, and $-\dfrac{1}{2} < -\dfrac{1}{3}$.

77. True. There are three possible cases.
Case 1: If $a > 0$ and $b > 0$, then $a^3 > b^3$, since $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right) > 0$.
Case 2: If $a > 0$ and $b < 0$, then $a^3 > 0$ and $b^3 < 0$, and it follows that $a^3 > b^3$.
Case 3: If $a < 0$ and $b < 0$, then $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right) > 0$, and we see that $a^3 > b^3$. (Note that $a - b > 0$ and $ab > 0$.)

78. True. If $a > b$, then it follows that $-a < -b$ because an inequality symbol is reversed when both sides of the inequality are multiplied by a negative number.

79. False. If we take $a = -2$, then $|-a| = |-(-2)| = |2| = 2 \neq a$.

80. True. If $b < 0$, then $b^2 > 0$, and $\left|b^2\right| = b^2$.

81. True. If $a - 4 < 0$, then $|a-4| = 4 - a = |4-a|$. If $a - 4 > 0$, then $|4-a| = a - 4 = |a-4|$.

82. False. If we let $a = -2$, then $|a+1| = |-2+1| = |-1| = 1 \neq |-2| + 1 = 3$.

83. False. If we take $a = 3$ and $b = -1$, then $|a+b| = |3-1| = 2 \neq |a| + |b| = 3 + 1 = 4$.

84. False. If we take $a = 3$ and $b = -1$, then $|a-b| = 4 \neq |a| - |b| = 3 - 1 = 2$.

85. If the car is driven in the city, then it can be expected to cover $(18.1)(20) = 362\ \frac{\text{miles}}{\text{gal}} \cdot \text{gal}$, or 362 miles, on a full tank. If the car is driven on the highway, then it can be expected to cover $(18.1)(27) = 488.7\ \frac{\text{miles}}{\text{gal}} \cdot \text{gal}$, or 488.7 miles, on a full tank. Thus, the driving range of the car may be described by the interval [362, 488.7].

86. Simplifying $5(C - 25) \geq 1.75 + 2.5C$, we obtain $5C - 125 \geq 1.75 + 2.5C$, $5C - 2.5C \geq 1.75 + 125$, $2.5C \geq 126.75$, and finally $C \geq 50.7$. Therefore, the minimum cost is \$50.70.

87. $6(P - 2500) \leq 4(P + 2400)$ can be rewritten as $6P - 15{,}000 \leq 4P + 9600$, $2P \leq 24{,}600$, or $P \leq 12{,}300$. Therefore, the maximum profit is \$12,300.

88. a. We want to find a formula for converting Centigrade temperatures to Fahrenheit temperatures. Thus, $C = \frac{5}{9}(F - 32) = \frac{5}{9}F - \frac{160}{9}$. Therefore, $\frac{5}{9}F = C + \frac{160}{9}$, $5F = 9C + 160$, and $F = \frac{9}{5}C + 32$. Calculating the lower temperature range, we have $F = \frac{9}{5}(-15) + 32 = 5$, or 5 degrees. Calculating the upper temperature range, $F = \frac{9}{5}(-5) + 32 = 23$, or 23 degrees. Therefore, the temperature range is $5^\circ < F < 23^\circ$.

b. For the lower temperature range, $C = \frac{5}{9}(63 - 32) = \frac{155}{9} \approx 17.2$, or 17.2 degrees. For the upper temperature range, $C = \frac{5}{9}(80 - 32) = \frac{5}{9}(48) \approx 26.7$, or 26.7 degrees. Therefore, the temperature range is $17.2^\circ < C < 26.7^\circ$.

89. Let x represent the salesman's monthly sales in dollars. Then $0.15(x - 12{,}000) \geq 6000$, $15(x - 12{,}000) \geq 600{,}000$, $15x - 180{,}000 \geq 600{,}000$, $15x \geq 780{,}000$, and $x \geq 52{,}000$. We conclude that the salesman must have sales of at least $52,000 to reach his goal.

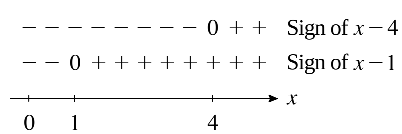
90. Let x represent the wholesale price of the car. Then $\dfrac{\text{Selling price}}{\text{Wholesale price}} - 1 \geq \text{Markup}$; that is, $\dfrac{11{,}200}{x} - 1 \geq 0.30$, whence $\dfrac{11{,}200}{x} \geq 1.30$, $1.3x \leq 11{,}200$, and $x \leq 8615.38$. We conclude that the maximum wholesale price is $8615.38.

91. The rod is acceptable if $0.49 \leq x \leq 0.51$ or $-0.01 \leq x - 0.5 \leq 0.01$. This gives the required inequality, $|x - 0.5| \leq 0.01$.

92. $|x - 0.1| \leq 0.01$ is equivalent to $-0.01 \leq x - 0.1 \leq 0.01$ or $0.09 \leq x \leq 0.11$. Therefore, the smallest diameter a ball bearing in the batch can have is 0.09 inch, and the largest diameter is 0.11 inch.

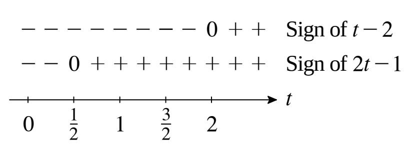
93. We want to solve the inequality $-6x^2 + 30x - 10 \geq 14$. (Remember that x is expressed in thousands.) Adding -14 to both sides of this inequality, we have $-6x^2 + 30x - 10 - 14 \geq 14 - 14$, or $-6x^2 + 30x - 24 \geq 0$. Dividing both sides of the inequality by -6 (which reverses the sign of the inequality), we have $x^2 - 5x + 4 \leq 0$. Factoring this last expression, we have $(x - 4)(x - 1) \leq 0$.

From the sign diagram, we see that x must lie between 1 and 4. (The inequality is satisfied only when the two factors have opposite signs.) Because x is expressed in thousands of units, we see that the manufacturer must produce between 1000 and 4000 units of the commodity.



94. We solve the inequality $\dfrac{0.2t}{t^2 + 1} \geq 0.08$, obtaining $0.08t^2 + 0.08 \leq 0.2t$, $0.08t^2 - 0.2t + 0.08 \leq 0$, $2t^2 - 5t + 2 \leq 0$, and $(2t - 1)(t - 2) \leq 0$.

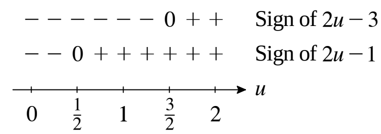
From the sign diagram, we see that the required solution is $\left[\frac{1}{2}, 2\right]$, so the concentration of the drug is greater than or equal to 0.08 mg/cc between $\frac{1}{2}$ hr and 2 hr after injection.



95. We solve the inequalities $25 \leq \dfrac{0.5x}{100 - x} \leq 30$, obtaining $2500 - 25x \leq 0.5x \leq 3000 - 30x$, which is equivalent to $2500 - 25x \leq 0.5x$ and $0.5x \leq 3000 - 30x$. Simplifying further, $25.5x \geq 2500$ and $30.5x \leq 3000$, so $x \geq \dfrac{2500}{25.5} \approx 98.04$ and $x \leq \dfrac{3000}{30.5} \approx 98.36$. Thus, the city could expect to remove between 98.04% and 98.36% of the toxic pollutant.

96. We simplify the inequality $20t - 40\sqrt{t} + 50 \le 35$ to $20t - 40\sqrt{t} + 15 \le 0$ (1). Let $u = \sqrt{t}$. Then $u^2 = t$, so we have $20u^2 - 40u + 15 \le 0$, $4u^2 - 8u + 3 \le 0$, and $(2u - 3)(2u - 1) \le 0$.

From the sign diagram, we see that we must have u in $\left[\frac{1}{2}, \frac{3}{2}\right]$.



Because $t = u^2$, we see that the solution to Equation (1) is $\left[\frac{1}{4}, \frac{9}{4}\right]$.

Thus, the average speed of a vehicle is less than or equal to 35 miles per hour between 6:15 a.m. and 8:15 a.m.

97. We solve the inequality $\dfrac{136}{1 + 0.25(t - 4.5)^2} + 28 \ge 128$ or $\dfrac{136}{1 + 0.25(t - 4.5)^2} \ge 100$. Next, $136 \ge 100\left[1 + 0.25(t - 4.5)^2\right]$, so $136 \ge 100 + 25(t - 4.5)^2$, $36 \ge 25(t - 4.5)^2$, $(t - 4.5)^2 \le \frac{36}{25}$, and $t - 4.5 \le \pm\frac{6}{5}$. Solving this last inequality, we have $t \le 5.7$ and $t \ge 3.3$. Thus, the amount of nitrogen dioxide is greater than or equal to 128 PSI between 10:18 a.m. and 12:42 p.m.

98. False. Take $a = 2$, $b = 3$, and $c = 4$. Then $\dfrac{a}{b + c} = \dfrac{2}{3 + 4} = \dfrac{2}{7}$, but $\dfrac{a}{b} + \dfrac{a}{c} = \dfrac{2}{3} + \dfrac{2}{4} = \dfrac{8 + 6}{12} = \dfrac{14}{12} = \dfrac{7}{6}$.

99. False. Take $a = 1$, $b = 2$, and $c = 3$. Then $a < b$, but $a - c = 1 - 3 = -2 \not> 2 - 3 = -1 = b - c$.

100. True. $|b - a| = |(-1)(a - b)| = |-1|\,|a - b| = |a - b|$.

101. True. $|a - b| = |a + (-b)| \le |a| + |-b| = |a| + |b|$.

102. False. Take $a = 3$ and $b = 1$. Then $\sqrt{a^2 - b^2} = \sqrt{9 - 1} = \sqrt{8} = 2\sqrt{2}$, but $|a| - |b| = 3 - 1 = 2$.

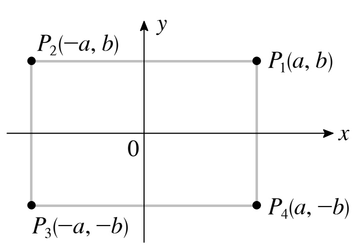
1.3 The Cartesian Coordinate System

Concept Questions

page 29

1. a. $a < 0$ and $b > 0$ **b.** $a < 0$ and $b < 0$ **c.** $a > 0$ and $b < 0$

2. a.



b. $d(P_1(a, b), (0, 0)) = \sqrt{(0 - a)^2 + (0 - b)^2} = \sqrt{a^2 + b^2}$,

$d(P_2(-a, b), (0, 0)) = \sqrt{[0 - (-a)]^2 + (0 - b)^2} = \sqrt{a^2 + b^2}$,

$d(P_3(-a, -b), (0, 0)) = \sqrt{[0 - (-a)]^2 + [0 - (-b)]^2} = \sqrt{a^2 + b^2}$,

and $d(P_4(a, -b), (0, 0)) = \sqrt{(0 - a)^2 + [0 - (-b)]^2} = \sqrt{a^2 + b^2}$,

so the points $P_1(a, b)$, $P_2(-a, b)$, $P_3(-a, -b)$, and $P_4(a, -b)$ are all the same distance from the origin.

Exercises

page 30

1. The coordinates of A are $(3, 3)$ and it is located in Quadrant I.

2. The coordinates of B are $(-5, 2)$ and it is located in Quadrant II.

3. The coordinates of C are $(2, -2)$ and it is located in Quadrant IV.

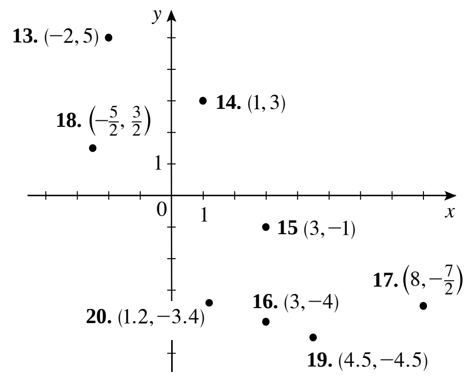
4. The coordinates of D are $(-2, 5)$ and it is located in Quadrant II.

5. The coordinates of E are $(-4, -6)$ and it is located in Quadrant III.

6. The coordinates of F are $(8, -2)$ and it is located in Quadrant IV.

7. A **8.** $(-5, 4)$ **9.** E, F, and G **10.** E **11.** F **12.** D

For Exercises 13–20, refer to the following figure.



21. Using the distance formula, we find that $\sqrt{(4-1)^2+(7-3)^2} = \sqrt{3^2+4^2} = \sqrt{25} = 5$.

22. Using the distance formula, we find that $\sqrt{(4-1)^2+(4-0)^2} = \sqrt{3^2+4^2} = \sqrt{25} = 5$.

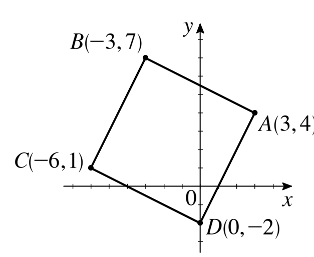
23. Using the distance formula, we find that $\sqrt{[4-(-1)]^2+(9-3)^2} = \sqrt{5^2+6^2} = \sqrt{25+36} = \sqrt{61}$.

24. Using the distance formula, we find that $\sqrt{[10-(-2)]^2+(6-1)^2} = \sqrt{12^2+5^2} = \sqrt{144+25} = \sqrt{169} = 13$.

25. The coordinates of the points have the form $(x, -6)$. Because the points are 10 units away from the origin, we have $(x-0)^2+(-6-0)^2 = 10^2$, $x^2 = 64$, or $x = \pm 8$. Therefore, the required points are $(-8, -6)$ and $(8, -6)$.

26. The coordinates of the points have the form $(3, y)$. Because the points are 5 units away from the origin, we have $(3-0)^2+(y-0)^2 = 5^2$, $y^2 = 16$, or $y = \pm 4$. Therefore, the required points are $(3, 4)$ and $(3, -4)$.

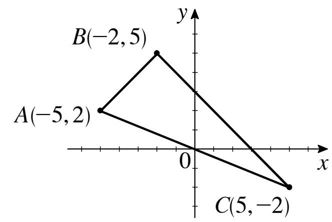
27. The points are shown in the diagram. To show that the four sides are equal, we compute



$d(A, B) = \sqrt{(-3-3)^2+(7-4)^2} = \sqrt{(-6)^2+3^2} = \sqrt{45}$,
$d(B, C) = \sqrt{[-6-(-3)]^2+(1-7)^2} = \sqrt{(-3)^2+(-6)^2} = \sqrt{45}$,
$d(C, D) = \sqrt{[0-(-6)]^2+[(-2)-1]^2} = \sqrt{(6)^2+(-3)^2} = \sqrt{45}$,
and $d(A, D) = \sqrt{(0-3)^2+(-2-4)^2} = \sqrt{(3)^2+(-6)^2} = \sqrt{45}$.

Next, to show that ΔABC is a right triangle, we show that it satisfies the Pythagorean Theorem. Thus, $d(A, C) = \sqrt{(-6-3)^2+(1-4)^2} = \sqrt{(-9)^2+(-3)^2} = \sqrt{90} = 3\sqrt{10}$ and $[d(A, B)]^2 + [d(B, C)]^2 = 90 = [d(A, C)]^2$. Similarly, $d(B, D) = \sqrt{90} = 3\sqrt{10}$, so ΔBAD is a right triangle as well. It follows that $\angle B$ and $\angle D$ are right angles, and we conclude that $ADCB$ is a square.

28. The triangle is shown in the figure. To prove that ΔABC is a right triangle, we show that $[d(A,C)]^2 = [d(A,B)]^2 + [d(B,C)]^2$ and the result will then follow from the Pythagorean Theorem. Now $[d(A,C)]^2 = (-5-5)^2 + [2-(-2)]^2 = 100 + 16 = 116$. Next, we find $[d(A,B)]^2 + [d(B,C)]^2 = [-2-(-5)]^2 + (5-2)^2 + [5-(-2)]^2 + (-2-5)^2 = 9 + 9 + 49 + 49 = 116$, and the result follows.



29. The equation of the circle with radius 5 and center $(2,-3)$ is given by $(x-2)^2 + [y-(-3)]^2 = 5^2$, or $(x-2)^2 + (y+3)^2 = 25$.

30. The equation of the circle with radius 3 and center $(-2,-4)$ is given by $[x-(-2)]^2 + [y-(-4)]^2 = 9$, or $(x+2)^2 + (y+4)^2 = 9$.

31. The equation of the circle with radius 5 and center $(0,0)$ is given by $(x-0)^2 + (y-0)^2 = 5^2$, or $x^2 + y^2 = 25$.

32. The distance between the center of the circle and the point $(2,3)$ on the circumference of the circle is given by $d = \sqrt{(3-0)^2 + (2-0)^2} = \sqrt{13}$. Therefore $r = \sqrt{13}$ and the equation of the circle centered at the origin that passes through $(2,3)$ is $x^2 + y^2 = 13$.

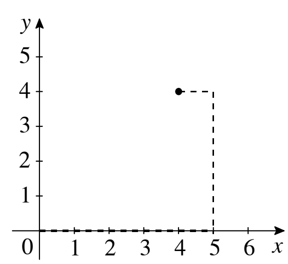
33. The distance between the points $(5,2)$ and $(2,-3)$ is given by $d = \sqrt{(5-2)^2 + [2-(-3)]^2} = \sqrt{3^2 + 5^2} = \sqrt{34}$. Therefore $r = \sqrt{34}$ and the equation of the circle passing through $(5,2)$ and $(2,-3)$ is $(x-2)^2 + [y-(-3)]^2 = 34$, or $(x-2)^2 + (y+3)^2 = 34$.

34. The equation of the circle with center $(-a, a)$ and radius $2a$ is given by $[x-(-a)]^2 + (y-a)^2 = (2a)^2$, or $(x+a)^2 + (y-a)^2 = 4a^2$.

35. **a.** The coordinates of the suspect's car at its final destination are $x = 4$ and $y = 4$.

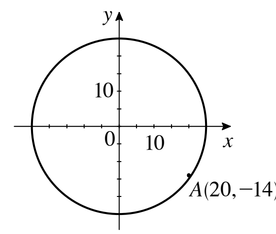
b. The distance traveled by the suspect was $5 + 4 + 1$, or 10 miles.

c. The distance between the original and final positions of the suspect's car was $d = \sqrt{(4-0)^2 + (4-0)^2} = \sqrt{32} = 4\sqrt{2}$, or approximately 5.66 miles.

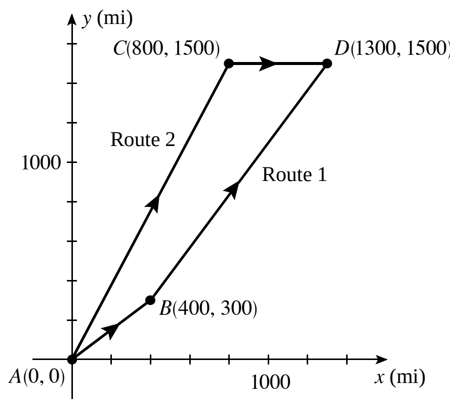


36. Referring to the diagram on page 31 of the text, we see that the distance from A to B is given by $d(A,B) = \sqrt{400^2 + 300^2} = \sqrt{250{,}000} = 500$. The distance from B to C is given by $d(B,C) = \sqrt{(-800-400)^2 + (800-300)^2} = \sqrt{(-1200)^2 + (500)^2} = \sqrt{1{,}690{,}000} = 1300$. The distance from C to D is given by $d(C,D) = \sqrt{[-800-(-800)]^2 + (800-0)^2} = \sqrt{0 + 800^2} = 800$. The distance from D to A is given by $d(D,A) = \sqrt{[(-800)-0]^2 + (0-0)} = \sqrt{640{,}000} = 800$. Therefore, the total distance covered on the tour is $d(A,B) + d(B,C) + d(C,D) + d(D,A) = 500 + 1300 + 800 + 800 = 3400$, or 3400 miles.

37. Suppose that the furniture store is located at the origin O so that your house is located at $A(20, -14)$. Because $d(O, A) = \sqrt{20^2 + (-14)^2} = \sqrt{596} \approx 24.4$, your house is located within a 25-mile radius of the store and you will not incur a delivery charge.



38.



Referring to the diagram, we see that the distance the salesman would cover if he took Route 1 is given by

$$d(A, B) + d(B, D) = \sqrt{400^2 + 300^2} + \sqrt{(1300 - 400)^2 + (1500 - 300)^2}$$
$$= \sqrt{250{,}000} + \sqrt{2{,}250{,}000} = 500 + 1500 = 2000$$

or 2000 miles. On the other hand, the distance he would cover if he took Route 2 is given by

$$d(A, C) + d(C, D) = \sqrt{800^2 + 1500^2} + \sqrt{(1300 - 800)^2} = \sqrt{2{,}890{,}000} + \sqrt{250{,}000}$$
$$= 1700 + 500 = 2200$$

or 2200 miles. Comparing these results, we see that he should take Route 1.

39. The cost of shipping by freight train is $(0.66)(2000)(100) = 132{,}000$, or \$132,000.
The cost of shipping by truck is $(0.62)(2200)(100) = 136{,}400$, or \$136,400.
Comparing these results, we see that the automobiles should be shipped by freight train. The net savings are $136{,}400 - 132{,}000 = 4400$, or \$4400.

40. The length of cable required on land is $d(S, Q) = 10{,}000 - x$ and the length of cable required under water is $d(Q, M) = \sqrt{\left(x^2 - 0\right) + (0 - 3000)^2} = \sqrt{x^2 + 3000^2}$. The cost of laying cable is thus $3(10{,}000 - x) + 5\sqrt{x^2 + 3000^2}$.
If $x = 2500$, then the total cost is given by $3(10{,}000 - 2500) + 5\sqrt{2500^2 + 3000^2} \approx 42{,}025.62$, or \$42,025.62.
If $x = 3000$, then the total cost is given by $3(10{,}000 - 3000) + 5\sqrt{3000^2 + 3000^2} \approx 42{,}213.20$, or \$42,213.20.

41. To determine the VHF requirements, we calculate $d = \sqrt{25^2 + 35^2} = \sqrt{625 + 1225} = \sqrt{1850} \approx 43.01$.
Models B, C, and D satisfy this requirement.
To determine the UHF requirements, we calculate $d = \sqrt{20^2 + 32^2} = \sqrt{400 + 1024} = \sqrt{1424} \approx 37.74$. Models C and D satisfy this requirement.
Therefore, Model C allows him to receive both channels at the least cost.

42. a. Let the positions of ships A and B after t hours be $A(0, y)$ and $B(x, 0)$, respectively. Then $x = 30t$ and $y = 20t$. Therefore, the distance in miles between the two ships is $D = \sqrt{(30t)^2 + (20t)^2} = \sqrt{900t^2 + 400t^2} = 10\sqrt{13}t$.

b. The required distance is obtained by letting $t = 2$, giving $D = 10\sqrt{13}(2)$, or approximately 72.11 miles.

43. a. Let the positions of ships A and B be $(0, y)$ and $(x, 0)$, respectively. Then $y = 25\left(t + \frac{1}{2}\right)$ and $x = 20t$. The distance D in miles between the two ships is

$$D = \sqrt{(x-0)^2 + (0-y)^2} = \sqrt{x^2 + y^2} = \sqrt{400t^2 + 625\left(t + \frac{1}{2}\right)^2} \quad (1).$$

b. The distance between the ships 2 hours after ship A has left port is obtained by letting $t = \frac{3}{2}$ in Equation (1), yielding $D = \sqrt{400\left(\frac{3}{2}\right)^2 + 625\left(\frac{3}{2} + \frac{1}{2}\right)^2} = \sqrt{3400}$, or approximately 58.31 miles.

44. a. The distance in feet is given by $\sqrt{(4000)^2 + x^2} = \sqrt{16{,}000{,}000 + x^2}$.

b. Substituting the value $x = 20{,}000$ into the above expression gives $\sqrt{16{,}000{,}000 + (20{,}000)^2} \approx 20{,}396$, or 20,396 ft.

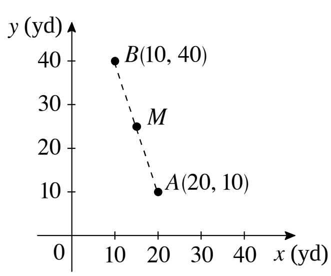
45. a. Suppose that $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are endpoints of the line segment and that the point $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is the midpoint of the line segment PQ. The distance between P and Q is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. The distance between P and M is

$$\sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2} = \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} = \tfrac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

which is one-half the distance from P to Q. Similarly, we obtain the same expression for the distance from M to P.

b. The midpoint is given by $\left(\dfrac{4-3}{2}, \dfrac{-5+2}{2}\right)$, or $\left(\dfrac{1}{2}, -\dfrac{3}{2}\right)$.

46. a.



b. The coordinates of the position of the prize are $x = \dfrac{20 + 10}{2}$ and $y = \dfrac{10 + 40}{2}$, or $x = 15$ yards and $y = 25$ yards.

c. The distance from the prize to the house is

$$d(M(15, 25), (0, 0)) = \sqrt{(15-0)^2 + (25-0)^2} = \sqrt{850}$$
$$\approx 29.15 \text{ (yards)}.$$

47. True. Plot the points.

48. True. Plot the points.

49. False. The distance between $P_1(a, b)$ and $P_3(kc, kd)$ is

$$d = \sqrt{(kc - a)^2 + (kd - b)^2}$$
$$\neq |k|\, D = |k| \sqrt{(c-a)^2 + (d-b)^2} = \sqrt{k^2(c-a)^2 + k^2(d-b)^2} = \sqrt{[k(c-a)]^2 + [k(d-b)]^2}.$$

50. True. $kx^2 + ky^2 = a^2$ gives $x^2 + y^2 = \dfrac{a^2}{k} < a^2$ if $k > 1$. So the radius of the circle with equation $kx^2 + ky^2 = a^2$ is a circle of radius smaller than a centered at the origin if $k > 1$. Therefore, it lies inside the circle of radius a with equation $x^2 + y^2 = a^2$.

51. Referring to the figure in the text, we see that the distance between the two points is given by the length of the hypotenuse of the right triangle. That is, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

52. $(x - h)^2 + (y - k)^2 = r^2$; $x^2 - 2xh + h^2 + y^2 - 2ky + k^2 = r^2$. This has the form $x^2 + y^2 + Cx + Dy + E = 0$, where $C = -2h$, $D = -2k$, and $E = h^2 + k^2 - r^2$.

1.4 Straight Lines

Concept Questions page 42

1. The slope is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $P(x_1, y_1)$ and $P(x_2, y_2)$ are any two distinct points on the nonvertical line. The slope of a vertical line is undefined.

2. a. $y - y_1 = m(x - x_1)$ **b.** $y = mx + b$ **c.** $ax + by + c = 0$, where a and b are not both zero.

3. a. $m_1 = m_2$ **b.** $m_2 = -\dfrac{1}{m_1}$

4. a. Solving the equation for y gives $By = -Ax - C$, so $y = -\dfrac{A}{B}x - \dfrac{C}{B}$. The slope of L is the coefficient of x, $-\dfrac{A}{B}$.

b. If $B = 0$, then the equation reduces to $Ax + C = 0$. Solving this equation for x, we obtain $x = -\dfrac{C}{A}$. This is an equation of a vertical line, and we conclude that the slope of L is undefined.

Exercises page 42

1. (e) **2.** (c) **3.** (a) **4.** (d) **5.** (f) **6.** (b)

7. Referring to the figure shown in the text, we see that $m = \dfrac{2 - 0}{0 - (-4)} = \dfrac{1}{2}$.

8. Referring to the figure shown in the text, we see that $m = \dfrac{4 - 0}{0 - 2} = -2$.

9. This is a vertical line, and hence its slope is undefined.

10. This is a horizontal line, and hence its slope is 0.

11. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 3}{5 - 4} = 5$.

12. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 5}{3 - 4} = \dfrac{3}{-1} = -3$.

13. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 3}{4 - (-2)} = \dfrac{5}{6}$.

14. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4 - (-2)}{4 - (-2)} = \dfrac{-2}{6} = -\dfrac{1}{3}$.

15. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{d - b}{c - a}$, provided $a \neq c$.

16. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-b - (b-1)}{a + 1 - (-a + 1)} = -\dfrac{-b - b + 1}{a + 1 + a - 1} = \dfrac{1 - 2b}{2a}$.

17. Because the equation is already in slope-intercept form, we read off the slope $m = 4$.

a. If x increases by 1 unit, then y increases by 4 units.

b. If x decreases by 2 units, then y decreases by $4(-2) = -8$ units.

18. Rewrite the given equation in slope-intercept form: $2x + 3y = 4$, $3y = 4 - 2x$, and so $y = \frac{4}{3} - \frac{2}{3}x$.

a. Because $m = -\frac{2}{3}$, we conclude that the slope is negative.

b. Because the slope is negative, y decreases as x increases.

c. If x decreases by 2 units, then y increases by $\left(-\frac{2}{3}\right)(-2) = \frac{4}{3}$ units.

19. The slope of the line through A and B is $\dfrac{-10 - (-2)}{-3 - 1} = \dfrac{-8}{-4} = 2$. The slope of the line through C and D is $\dfrac{1 - 5}{-1 - 1} = \dfrac{-4}{-2} = 2$. Because the slopes of these two lines are equal, the lines are parallel.

20. The slope of the line through A and B is $\dfrac{-2 - 3}{2 - 2}$. Because this slope is undefined, we see that the line is vertical. The slope of the line through C and D is $\dfrac{5 - 4}{-2 - (-2)}$. Because this slope is undefined, we see that this line is also vertical. Therefore, the lines are parallel.

21. The slope of the line through A and B is $\dfrac{2 - 5}{4 - (-2)} = -\dfrac{3}{6} = -\dfrac{1}{2}$. The slope of the line through C and D is $\dfrac{6 - (-2)}{3 - (-1)} = \dfrac{8}{4} = 2$. Because the slopes of these two lines are the negative reciprocals of each other, the lines are perpendicular.

22. The slope of the line through A and B is $\dfrac{-2 - 0}{1 - 2} = \dfrac{-2}{-1} = 2$. The slope of the line through C and D is $\dfrac{4 - 2}{-8 - 4} = \dfrac{2}{-12} = -\dfrac{1}{6}$. Because the slopes of these two lines are not the negative reciprocals of each other, the lines are not perpendicular.

23. The slope of the line through the point $(1, a)$ and $(4, -2)$ is $m_1 = \dfrac{-2 - a}{4 - 1}$ and the slope of the line through $(2, 8)$ and $(-7, a + 4)$ is $m_2 = \dfrac{a + 4 - 8}{-7 - 2}$. Because these two lines are parallel, m_1 is equal to m_2. Therefore, $\dfrac{-2 - a}{3} = \dfrac{a - 4}{-9}$, $-9(-2 - a) = 3(a - 4)$, $18 + 9a = 3a - 12$, and $6a = -30$, so $a = -5$.

24. The slope of the line through the point $(a, 1)$ and $(5, 8)$ is $m_1 = \dfrac{8 - 1}{5 - a}$ and the slope of the line through $(4, 9)$ and $(a + 2, 1)$ is $m_2 = \dfrac{1 - 9}{a + 2 - 4}$. Because these two lines are parallel, m_1 is equal to m_2. Therefore, $\dfrac{7}{5 - a} = \dfrac{-8}{a - 2}$, $7(a - 2) = -8(5 - a)$, $7a - 14 = -40 + 8a$, and $a = 26$.

25. An equation of a horizontal line is of the form $y = b$. In this case $b = -3$, so $y = -3$ is an equation of the line.

26. An equation of a vertical line is of the form $x = a$. In this case $a = 0$, so $x = 0$ is an equation of the line.

27. We use the point-slope form of an equation of a line with the point $(3, -4)$ and slope $m = 2$. Thus $y - y_1 = m(x - x_1)$ becomes $y - (-4) = 2(x - 3)$. Simplifying, we have $y + 4 = 2x - 6$, or $y = 2x - 10$.

28. We use the point-slope form of an equation of a line with the point $(2, 4)$ and slope $m = -1$. Thus $y - y_1 = m(x - x_1)$, giving $y - 4 = -1(x - 2)$, $y - 4 = -x + 2$, and finally $y = -x + 6$.

29. Because the slope $m = 0$, we know that the line is a horizontal line of the form $y = b$. Because the line passes through $(-3, 2)$, we see that $b = 2$, and an equation of the line is $y = 2$.

30. We use the point-slope form of an equation of a line with the point $(1, 2)$ and slope $m = -\frac{1}{2}$. Thus $y - y_1 = m(x - x_1)$ gives $y - 2 = -\frac{1}{2}(x - 1)$, $2y - 4 = -x + 1$, $2y = -x + 5$, and $y = -\frac{1}{2}x + \frac{5}{2}$.

31. We first compute the slope of the line joining the points $(2, 4)$ and $(3, 7)$ to be $m = \dfrac{7 - 4}{3 - 2} = 3$. Using the point-slope form of an equation of a line with the point $(2, 4)$ and slope $m = 3$, we find $y - 4 = 3(x - 2)$, or $y = 3x - 2$.

32. We first compute the slope of the line joining the points $(2, 1)$ and $(2, 5)$ to be $m = \dfrac{5 - 1}{2 - 2}$. Because this slope is undefined, we see that the line must be a vertical line of the form $x = a$. Because it passes through $(2, 5)$, we see that $x = 2$ is the equation of the line.

33. We first compute the slope of the line joining the points $(1, 2)$ and $(-3, -2)$ to be $m = \dfrac{-2 - 2}{-3 - 1} = \dfrac{-4}{-4} = 1$. Using the point-slope form of an equation of a line with the point $(1, 2)$ and slope $m = 1$, we find $y - 2 = x - 1$, or $y = x + 1$.

34. We first compute the slope of the line joining the points $(-1, -2)$ and $(3, -4)$ to be $m = \dfrac{-4 - (-2)}{3 - (-1)} = \dfrac{-2}{4} = -\dfrac{1}{2}$. Using the point-slope form of an equation of a line with the point $(-1, -2)$ and slope $m = -\frac{1}{2}$, we find $y - (-2) = -\frac{1}{2}[x - (-1)]$, $y + 2 = -\frac{1}{2}(x + 1)$, and finally $y = -\frac{1}{2}x - \frac{5}{2}$.

35. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = 3$ and $b = 4$, the equation is $y = 3x + 4$.

36. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = -2$ and $b = -1$, the equation is $y = -2x - 1$.

37. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = 0$ and $b = 5$, the equation is $y = 5$.

38. We use the slope-intercept form of an equation of a line:$y = mx + b$. Because $m = -\frac{1}{2}$, and $b = \frac{3}{4}$, the equation is $y = -\frac{1}{2}x + \frac{3}{4}$.

39. We first write the given equation in the slope-intercept form: $x - 2y = 0$, so $-2y = -x$, or $y = \frac{1}{2}x$. From this equation, we see that $m = \frac{1}{2}$ and $b = 0$.

40. We write the equation in slope-intercept form: $y - 2 = 0$, so $y = 2$. From this equation, we see that $m = 0$ and $b = 2$.

41. We write the equation in slope-intercept form: $2x - 3y - 9 = 0$, $-3y = -2x + 9$, and $y = \frac{2}{3}x - 3$. From this equation, we see that $m = \frac{2}{3}$ and $b = -3$.

42. We write the equation in slope-intercept form: $3x - 4y + 8 = 0$, $-4y = -3x - 8$, and $y = \frac{3}{4}x + 2$. From this equation, we see that $m = \frac{3}{4}$ and $b = 2$.

43. We write the equation in slope-intercept form: $2x + 4y = 14$, $4y = -2x + 14$, and $y = -\frac{2}{4}x + \frac{14}{4} = -\frac{1}{2}x + \frac{7}{2}$. From this equation, we see that $m = -\frac{1}{2}$ and $b = \frac{7}{2}$.

44. We write the equation in the slope-intercept form: $5x + 8y - 24 = 0$, $8y = -5x + 24$, and $y = -\frac{5}{8}x + 3$. From this equation, we conclude that $m = -\frac{5}{8}$ and $b = 3$.

45. We first write the equation $2x - 4y - 8 = 0$ in slope-intercept form: $2x - 4y - 8 = 0$, $4y = 2x - 8$, $y = \frac{1}{2}x - 2$. Now the required line is parallel to this line, and hence has the same slope. Using the point-slope form of an equation of a line with $m = \frac{1}{2}$ and the point $(-2, 2)$, we have $y - 2 = \frac{1}{2}[x - (-2)]$ or $y = \frac{1}{2}x + 3$.

46. We first write the equation $3x + 4y - 22 = 0$ in slope-intercept form: $3x + 4y - 22 = 0$, so $4y = -3x + 22$ and $y = -\frac{3}{4}x + \frac{11}{2}$ Now the required line is perpendicular to this line, and hence has slope $\frac{4}{3}$ (the negative reciprocal of $-\frac{3}{4}$). Using the point-slope form of an equation of a line with $m = \frac{4}{3}$ and the point $(2, 4)$, we have $y - 4 = \frac{4}{3}(x - 2)$, or $y = \frac{4}{3}x + \frac{4}{3}$.

47. The midpoint of the line segment joining $P_1(-2, -4)$ and $P_2(3, 6)$ is $M\left(\dfrac{-2+3}{2}, \dfrac{-4+6}{2}\right)$ or $M\left(\frac{1}{2}, 1\right)$.

Using the point-slope form of the equation of a line with $m = -2$, we have $y - 1 = -2\left(x - \frac{1}{2}\right)$ or $y = -2x + 2$.

48. The midpoint of the line segment joining $P_1(-1, -3)$ and $P_2(3, 3)$ is $M_1\left(\dfrac{-1+3}{2}, \dfrac{-3+3}{2}\right)$ or $M_1(1, 0)$.

The midpoint of the line segment joining $P_3(-2, 3)$ and $P_4(2, -3)$ is $M_2\left(\dfrac{-2+2}{2}, \dfrac{3-3}{2}\right)$ or $M_2(0, 0)$.

The slope of the required line is $m = \dfrac{0-0}{1-0} = 0$, so an equation of the line is $y - 0 = 0(x - 0)$ or $y = 0$.

49. A line parallel to the x-axis has slope 0 and is of the form $y = b$. Because the line is 6 units below the axis, it passes through $(0, -6)$ and its equation is $y = -6$.

50. Because the required line is parallel to the line joining $(2, 4)$ and $(4, 7)$, it has slope $m = \dfrac{7-4}{4-2} = \dfrac{3}{2}$. We also know that the required line passes through the origin $(0, 0)$. Using the point-slope form of an equation of a line, we find $y - 0 = \frac{3}{2}(x - 0)$, or $y = \frac{3}{2}x$.

51. We use the point-slope form of an equation of a line to obtain $y - b = 0(x - a)$, or $y = b$.

52. Because the line is parallel to the x-axis, its slope is 0 and its equation has the form $y = b$. We know that the line passes through $(-3, 4)$, so the required equation is $y = 4$.

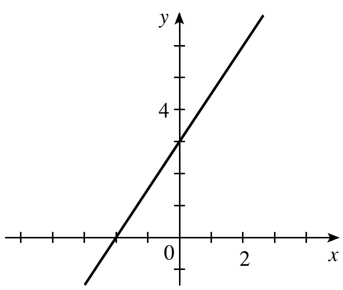
53. Because the required line is parallel to the line joining $(-3, 2)$ and $(6, 8)$, it has slope $m = \dfrac{8-2}{6-(-3)} = \dfrac{6}{9} = \dfrac{2}{3}$. We also know that the required line passes through $(-5, -4)$. Using the point-slope form of an equation of a line, we find $y - (-4) = \frac{2}{3}[x - (-5)]$, $y = \frac{2}{3}x + \frac{10}{3} - 4$, and finally $y = \frac{2}{3}x - \frac{2}{3}$.

54. Because the slope of the line is undefined, it has the form $x = a$. Furthermore, since the line passes through (a, b), the required equation is $x = a$.

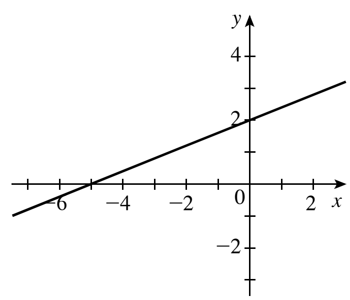
55. Because the point $(-3, 5)$ lies on the line $kx + 3y + 9 = 0$, it satisfies the equation. Substituting $x = -3$ and $y = 5$ into the equation gives $-3k + 15 + 9 = 0$, or $k = 8$.

56. Because the point $(2, -3)$ lies on the line $-2x + ky + 10 = 0$, it satisfies the equation. Substituting $x = 2$ and $y = -3$ into the equation gives $-2(2) + (-3)k + 10 = 0$, $-4 - 3k + 10 = 0$, $-3k = -6$, and finally $k = 2$.

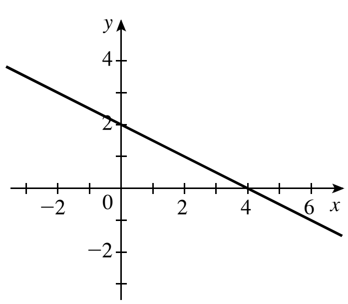
57. $3x - 2y + 6 = 0$. Setting $y = 0$, we have $3x + 6 = 0$ or $x = -2$, so the x-intercept is -2. Setting $x = 0$, we have $-2y + 6 = 0$ or $y = 3$, so the y-intercept is 3.



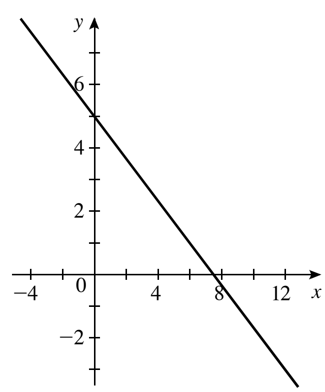
58. $2x - 5y + 10 = 0$. Setting $y = 0$, we have $2x + 10 = 0$ or $x = -5$, so the x-intercept is -5. Setting $x = 0$, we have $-5y + 10 = 0$ or $y = 2$, so the y-intercept is 2.



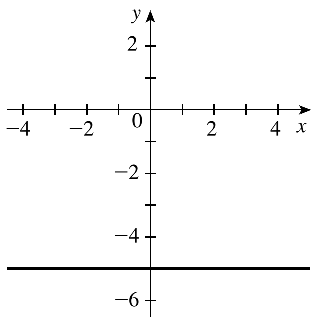
59. $x + 2y - 4 = 0$. Setting $y = 0$, we have $x - 4 = 0$ or $x = 4$, so the x-intercept is 4. Setting $x = 0$, we have $2y - 4 = 0$ or $y = 2$, so the y-intercept is 2.



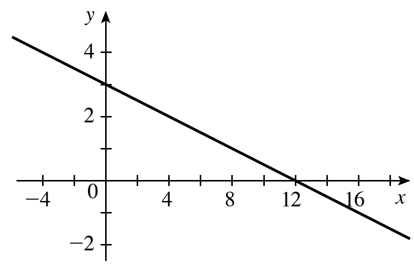
60. $2x + 3y - 15 = 0$. Setting $y = 0$, we have $2x - 15 = 0$, so the x-intercept is $\frac{15}{2}$. Setting $x = 0$, we have $3y - 15 = 0$, so the y-intercept is 5.



61. $y + 5 = 0$. Setting $y = 0$, we have $0 + 5 = 0$, which has no solution, so there is no x-intercept. Setting $x = 0$, we have $y + 5 = 0$ or $y = -5$, so the y-intercept is -5.



62. $-2x - 8y + 24 = 0$. Setting $y = 0$, we have $-2x + 24 = 0$ or $x = 12$, so the x-intercept is 12. Setting $x = 0$, we have $-8y + 24 = 0$ or $y = 3$, so the y-intercept is 3.



63. Because the line passes through the points $(a, 0)$ and $(0, b)$, its slope is $m = \dfrac{b-0}{0-a} = -\dfrac{b}{a}$. Then, using the point-slope form of an equation of a line with the point $(a, 0)$, we have $y - 0 = -\dfrac{b}{a}(x - a)$ or $y = -\dfrac{b}{a}x + b$, which may be written in the form $\dfrac{b}{a}x + y = b$. Multiplying this last equation by $\dfrac{1}{b}$, we have $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

64. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = 3$ and $b = 4$, we have $\dfrac{x}{3} + \dfrac{y}{4} = 1$. Then $4x + 3y = 12$, so $3y = 12 - 4x$ and thus $y = -\frac{4}{3}x + 4$.

65. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = -2$ and $b = -4$, we have $-\dfrac{x}{2} - \dfrac{y}{4} = 1$. Then $-4x - 2y = 8$, $2y = -8 - 4x$, and finally $y = -2x - 4$.

66. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = -\frac{1}{2}$ and $b = \frac{3}{4}$, we have $\dfrac{x}{-1/2} + \dfrac{y}{3/4} = 1$, $\frac{3}{4}x - \frac{1}{2}y = \left(-\frac{1}{2}\right)\left(\frac{3}{4}\right)$, $\frac{1}{2}y = -\frac{3}{4}x - \frac{3}{8}$, and finally $y = 2\left(\frac{3}{4}x + \frac{3}{8}\right) = \frac{3}{2}x + \frac{3}{4}$.

67. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = 4$ and $b = -\frac{1}{2}$, we have $\dfrac{x}{4} + \dfrac{y}{-1/2} = 1$, $-\frac{1}{4}x + 2y = -1$, $2y = \frac{1}{4}x - 1$, and so $y = \frac{1}{8}x - \frac{1}{2}$.

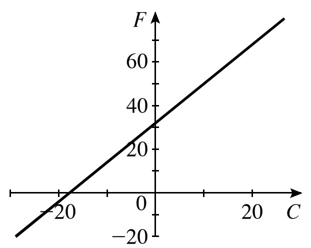
68. The slope of the line passing through A and B is $m = \dfrac{-2-7}{2-(-1)} = -\dfrac{9}{3} = -3$, and the slope of the line passing through B and C is $m = \dfrac{-9-(-2)}{5-2} = -\dfrac{7}{3}$. Because the slopes are not equal, the points do not lie on the same line.

69. The slope of the line passing through A and B is $m = \dfrac{7-1}{1-(-2)} = \dfrac{6}{3} = 2$, and the slope of the line passing through B and C is $m = \dfrac{13-7}{4-1} = \dfrac{6}{3} = 2$. Because the slopes are equal, the points lie on the same line.

70. The slope of the line L passing through $P_1(1.2, -9.04)$ and $P_2(2.3, -5.96)$ is $m = \dfrac{-5.96 - (-9.04)}{2.3 - 1.2} = 2.8$, so an equation of L is $y - (-9.04) = 2.8(x - 1.2)$ or $y = 2.8x - 12.4$.
Substituting $x = 4.8$ into this equation gives $y = 2.8(4.8) - 12.4 = 1.04$. This shows that the point $P_3(4.8, 1.04)$ lies on L. Next, substituting $x = 7.2$ into the equation gives $y = 2.8(7.2) - 12.4 = 7.76$, which shows that the point $P_4(7.2, 7.76)$ also lies on L. We conclude that John's claim is valid.

71. The slope of the line L passing through $P_1(1.8, -6.44)$ and $P_2(2.4, -5.72)$ is $m = \dfrac{-5.72 - (-6.44)}{2.4 - 1.8} = 1.2$, so an equation of L is $y - (-6.44) = 1.2(x - 1.8)$ or $y = 1.2x - 8.6$.
Substituting $x = 5.0$ into this equation gives $y = 1.2(5) - 8.6 = -2.6$. This shows that the point $P_3(5.0, -2.72)$ does not lie on L, and we conclude that Alison's claim is not valid.

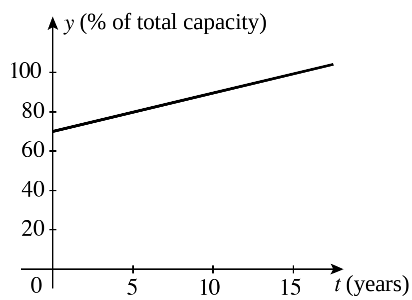
72. a.



b. The slope is $\frac{9}{5}$. It represents the change in °F per unit change in °C.

c. The F-intercept of the line is 32. It corresponds to 0°, so it is the freezing point in °F.

73. a.



b. The slope is 1.9467 and the y-intercept is 70.082.

c. The output is increasing at the rate of 1.9467% per year. The output at the beginning of 1990 was 70.082%.

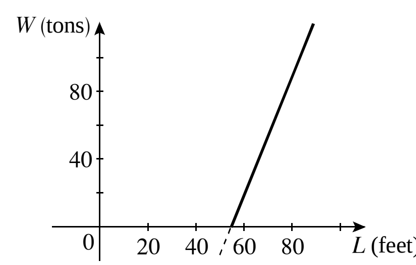
d. We solve the equation $1.9467t + 70.082 = 100$, obtaining $t \approx 15.37$. We conclude that the plants were generating at maximum capacity during April 2005.

74. a. $y = 0.0765x$ **b.** \$0.0765 **c.** $0.0765(65{,}000) = 4972.50$, or \$4972.50.

75. a. $y = 0.55x$ **b.** Solving the equation $1100 = 0.55x$ for x, we have $x = \dfrac{1100}{0.55} = 2000$.

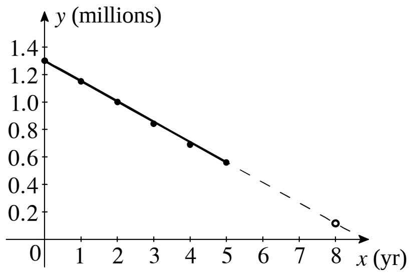
76. a. Substituting $L = 80$ into the given equation, we have $W = 3.51(80) - 192 = 280.8 - 192 = 88.8$, or 88.8 British tons.

b.



77. Using the points $(0, 0.68)$ and $(10, 0.80)$, we see that the slope of the required line is $m = \dfrac{0.80 - 0.68}{10 - 0} = \dfrac{0.12}{10} = 0.012$. Next, using the point-slope form of the equation of a line, we have $y - 0.68 = 0.012(t - 0)$ or $y = 0.012t + 0.68$. Therefore, when $t = 18$, we have $y = 0.012(18) + 0.68 = 0.896$, or 89.6%. That is, in 2008 women's wages were expected to be 89.6% of men's wages.

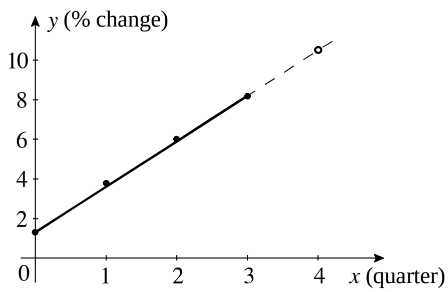
78. a, b.



c. The slope of L is $m = \dfrac{0.56 - 1.30}{5 - 0} = -0.148$, so an equation of L is $y - 1.3 = -0.148\,(x - 0)$ or $y = -0.148x + 1.3$.

d. The number of pay phones in 2012 is estimated to be $-0.148\,(8) + 1.3$, or approximately 116,000.

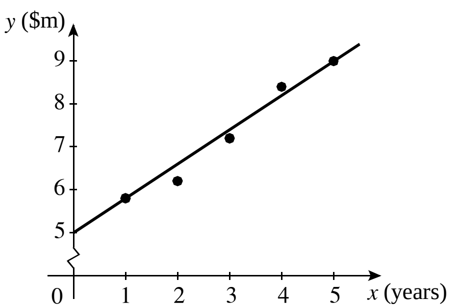
79. a, b.



c. The slope of L is $m = \dfrac{8.2 - 1.3}{3 - 0} = 2.3$, so an equation of L is $y - 1.3 = 2.3\,(x - 0)$ or $y = 2.3x + 1.3$.

d. The change in spending in the first quarter of 2014 is estimated to be $2.3\,(4) + 1.3$, or 10.5%.

80. a, b.

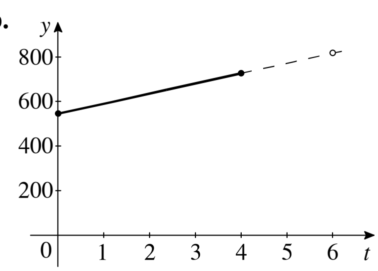


c. The slope of L is $m = \dfrac{9.0 - 5.8}{5 - 1} = \dfrac{3.2}{4} = 0.8$. Using the point-slope form of an equation of a line, we have $y - 5.8 = 0.8\,(x - 1) = 0.8x - 0.8$, or $y = 0.8x + 5$.

d. Using the equation from part (c) with $x = 9$, we have $y = 0.8\,(9) + 5 = 12.2$, or \$12.2 million.

81. a. The slope of the line L passing through $A\,(0, 545)$ and $B\,(4, 726)$ is $m = \dfrac{726 - 545}{4 - 0} = \dfrac{181}{4}$, so an equation of L is $y - 545 = \frac{181}{4}\,(t - 0)$ or $y = \frac{181}{4}t + 545$.

b.



c. The number of corporate fraud cases pending at the beginning of 2014 is estimated to be $\frac{181}{4}\,(6) + 545$, or approximately 817.

82. a. The slope of the line through $P_1\,(0, 27)$ and $P_2\,(1, 29)$ is $m_1 = \dfrac{29 - 27}{1 - 0} = 2$, which is equal to the slope of the line through $P_2\,(1, 29)$ and $P_3\,(2, 31)$, which is $m_2 = \dfrac{31 - 29}{1 - 0} = 2$. Thus, the three points lie on the line L.

b. The percentage is of moviegoers who use social media to chat about movies in 2014 is estimated to be $31 + 2\,(2)$, or 35%.

c. $y - 27 = 2\,(x - 0)$, so $y = 2x + 27$. The estimate for 2014 ($t = 4$) is $2\,(4) + 27 = 35$, as found in part (b).

83. True. The slope of the line is given by $-\frac{2}{4} = -\frac{1}{2}$.

84. True. The slope of the line $Ax + By + C = 0$ is $-\dfrac{A}{B}$. (Write it in slope-intercept form.) Similarly, the slope of the line $ax + by + c = 0$ is $-\dfrac{a}{b}$. They are parallel if and only if $-\dfrac{A}{B} = -\dfrac{a}{b}$, that is, if $Ab = aB$, or $Ab - aB = 0$.

85. False. Let the slope of L_1 be $m_1 > 0$. Then the slope of L_2 is $m_2 = -\dfrac{1}{m_1} < 0$.

86. True. The slope of the line $ax + by + c_1 = 0$ is $m_1 = -\dfrac{a}{b}$. The slope of the line $bx - ay + c_2 = 0$ is $m_2 = \dfrac{b}{a}$. Because $m_1 m_2 = -1$, the straight lines are indeed perpendicular.

87. True. Set $y = 0$ and we have $Ax + C = 0$ or $x = -C/A$, and this is where the line intersects the x-axis.

88. Yes. A straight line with slope zero ($m = 0$) is a horizontal line, whereas a straight line whose slope does not exist is a vertical line (m cannot be computed).

89. Writing each equation in the slope-intercept form, we have $y = -\dfrac{a_1}{b_1}x - \dfrac{c_1}{b_1}$ ($b_1 \neq 0$) and $y = -\dfrac{a_2}{b_2}x - \dfrac{c_2}{b_2}$ ($b_2 \neq 0$). Because two lines are parallel if and only if their slopes are equal, we see that the lines are parallel if and only if $-\dfrac{a_1}{b_1} = -\dfrac{a_2}{b_2}$, or $a_1 b_2 - b_1 a_2 = 0$.

90. The slope of L_1 is $m_1 = \dfrac{b - 0}{1 - 0} = b$. The slope of L_2 is $m_2 = \dfrac{c - 0}{1 - 0} = c$. Applying the Pythagorean theorem to ΔOAC and ΔOCB gives $(OA)^2 = 1^2 + b^2$ and $(OB)^2 = 1^2 + c^2$. Adding these equations and applying the Pythagorean theorem to ΔOBA gives $(AB)^2 = (OA)^2 + (OB)^2 = 1^2 + b^2 + 1^2 + c^2 = 2 + b^2 + c^2$. Also, $(AB)^2 = (b - c)^2$, so $(b - c)^2 = 2 + b^2 + c^2$, $b^2 - 2bc + c^2 = 2 + b^2 + c^2$, and $-2bc = 2$, $1 = -bc$. Finally, $m_1 m_2 = b \cdot c = bc = -1$, as was to be shown.

CHAPTER 1 Concept Review Questions page 48

1. ordered, abscissa or x-coordinate, ordinate or y-coordinate

2. a. x, y **b.** third

3. $\sqrt{(c-a)^2+(d-b)^2}$

4. $(x-a)^2+(y-b)^2=r^2$

5. a. $\dfrac{y_2-y_1}{x_2-x_1}$ **b.** undefined **c.** 0 **d.** positive

6. $m_1=m_2, m_1=-\dfrac{1}{m_2}$

7. a. $y-y_1=m\,(x-x_1)$, point-slope form **b.** $y=mx+b$, slope-intercept

8. a. $Ax+By+C=0$, where A and B are not both zero **b.** $-a/b$

CHAPTER 1 Review Exercises page 48

1. Adding x to both sides yields $3 \le 3x + 9$, $3x \ge -6$, or $x \ge -2$. We conclude that the solution set is $[-2, \infty)$.

2. $-2 \le 3x + 1 \le 7$ implies $-3 \le 3x \le 6$, or $-1 \le x \le 2$, and so the solution set is $[-1, 2]$.

3. The inequalities imply $x > 5$ or $x < -4$, so the solution set is $(-\infty, -4) \cup (5, \infty)$.

4. $2x^2 > 50$ is equivalent to $x^2 > 25$, so either $x > 5$ or $x < -5$ and the solution set is $(-\infty, -5) \cup (5, \infty)$.

5. $|-5+7| + |-2| = |2| + |-2| = 2 + 2 = 4.$

6. $\left|\dfrac{5-12}{-4-3}\right| = \dfrac{|5-12|}{|-7|} = \dfrac{|-7|}{7} = \dfrac{7}{7} = 1.$

7. $|2\pi - 6| - \pi = 2\pi - 6 - \pi = \pi - 6.$

8. $\left|\sqrt{3} - 4\right| + \left|4 - 2\sqrt{3}\right| = \left(4 - \sqrt{3}\right) + \left(4 - 2\sqrt{3}\right)$
$= 8 - 3\sqrt{3}.$

9. $\left(\dfrac{9}{4}\right)^{3/2} = \dfrac{9^{3/2}}{4^{3/2}} = \dfrac{27}{8}.$

10. $\dfrac{5^6}{5^4} = 5^{6-4} = 5^2 = 25.$

11. $(3 \cdot 4)^{-2} = 12^{-2} = \dfrac{1}{12^2} = \dfrac{1}{144}.$

12. $(-8)^{5/3} = \left(-8^{1/3}\right)^5 = (-2)^5 = -32.$

13. $\dfrac{\left(3 \cdot 2^{-3}\right)\left(4 \cdot 3^5\right)}{2 \cdot 9^3} = \dfrac{3 \cdot 2^{-3} \cdot 2^2 \cdot 3^5}{2 \cdot \left(3^2\right)^3} = \dfrac{2^{-1} \cdot 3^6}{2 \cdot 3^6} = \dfrac{1}{4}.$

14. $\dfrac{3\sqrt[3]{54}}{\sqrt[3]{18}} = \dfrac{3 \cdot \left(2 \cdot 3^3\right)^{1/3}}{\left(2 \cdot 3^2\right)^{1/3}} = \dfrac{3^2 \cdot 2^{1/3}}{2^{1/3} \cdot 3^{2/3}} = 3^{4/3} = 3\sqrt[3]{3}.$

15. $\dfrac{4\left(x^2 + y\right)^3}{x^2 + y} = 4\left(x^2 + y\right)^2.$

16. $\dfrac{a^6 b^{-5}}{\left(a^3 b^{-2}\right)^{-3}} = \dfrac{a^6 b^{-5}}{a^{-9} b^6} = \dfrac{a^{15}}{b^{11}}.$

17. $\dfrac{\sqrt[4]{16x^5yz}}{\sqrt[4]{81xyz^5}} = \dfrac{\left(2^4 x^5 yz\right)^{1/4}}{\left(3^4 xyz^5\right)^{1/4}} = \dfrac{2x^{5/4} y^{1/4} z^{1/4}}{3x^{1/4} y^{1/4} z^{5/4}} = \dfrac{2x}{3z}.$

18. $\left(2x^3\right)\left(-3x^{-2}\right)\left(\frac{1}{6}x^{-1/2}\right) = -x^{1/2}.$

19. $\left(\dfrac{3xy^2}{4x^3y}\right)^{-2}\left(\dfrac{3xy^3}{2x^2}\right)^3 = \left(\dfrac{3y}{4x^2}\right)^{-2}\left(\dfrac{3y^3}{2x}\right)^3 = \left(\dfrac{4x^2}{3y}\right)^2\left(\dfrac{3y^3}{2x}\right)^3 = \dfrac{\left(16x^4\right)\left(27y^9\right)}{\left(9y^2\right)\left(8x^3\right)} = 6xy^7.$

20. $\sqrt[3]{81x^5y^{10}}\sqrt[3]{9xy^2} = \sqrt[3]{\left(3^4x^5y^{10}\right)\left(3^2xy^2\right)} = \left(3^6x^6y^{12}\right)^{1/3} = 3^2x^2y^4 = 9x^2y^4.$

21. $-2\pi^2 r^3 + 100\pi r^2 = -2\pi r^2 (\pi r - 50).$

22. $2v^3w + 2vw^3 + 2u^2vw = 2vw\left(v^2 + w^2 + u^2\right).$

23. $16 - x^2 = 4^2 - x^2 = (4 - x)(4 + x).$

24. $12t^3 - 6t^2 - 18t = 6t\left(2t^2 - t - 3\right) = 6t(2t - 3)(t + 1).$

25. $8x^2 + 2x - 3 = (4x + 3)(2x - 1) = 0$, so $x = -\frac{3}{4}$ and $x = \frac{1}{2}$ are the roots of the equation.

26. $-6x^2 - 10x + 4 = 0$, $3x^2 + 5x - 2 = (3x - 1)(x + 2) = 0$, and so $x = -2$ or $x = \frac{1}{3}$.

27. $-x^3 - 2x^2 + 3x = -x\left(x^2 + 2x - 3\right) = -x\,(x+3)\,(x-1) = 0$, and so the roots of the equation are $x = 0$, $x = -3$, and $x = 1$.

28. $2x^4 + x^2 = 1$. If we let $y = x^2$, we can write the equation as $2y^2 + y - 1 = (2y-1)\,(y+1) = 0$, giving $y = \frac{1}{2}$ or $y = -1$. We reject the second root since $y = x^2$ must be nonnegative. Therefore, $x^2 = \frac{1}{2}$, and so $x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$.

29. Factoring the given expression, we have $(2x-1)\,(x+2) \le 0$. From the sign diagram, we conclude that the given inequality is satisfied when $-2 \le x \le \frac{1}{2}$.

$- - - - - - - \; 0 \; + \; +$ Sign of $2x-1$

$- - \; 0 \; + + + + + + +$ Sign of $x+2$

x: -2, 0, $\frac{1}{2}$

30. $\dfrac{1}{x+2} > 2$ gives $\dfrac{1}{x+2} - 2 > 0$, $\dfrac{1-2x-4}{x+2} > 0$, and finally $\dfrac{-2x-3}{x+2} > 0$. From the sign diagram, we see that the given inequality is satisfied when $-2 < x < -\frac{3}{2}$.

$+ + + + \; 0 \; - - - - - -$ Sign of $-2x-3$

$- - \; 0 \; + + + + + + + +$ Sign of $x+2$

x: -2, $-\frac{3}{2}$, 0

31. The given inequality is equivalent to $|2x-3| < 5$ or $-5 < 2x - 3 < 5$. Thus, $-2 < 2x < 8$, or $-1 < x < 4$.

32. The given equation implies that either $\dfrac{x+1}{x-1} = 5$ or $\dfrac{x+1}{x-1} = -5$. Solving the first equality, we have $x + 1 = 5\,(x-1) = 5x - 5$, $-4x = -6$, and $x = \frac{3}{2}$. Similarly, we solve the second equality and obtain $x + 1 = -5\,(x-1) = -5x + 5$, $6x = 4$, and $x = \frac{2}{3}$. Thus, the two values of x that satisfy the equation are $x = \frac{3}{2}$ and $x = \frac{2}{3}$.

33. We use the quadratic formula to solve the equation $x^2 - 2x - 5 = 0$. Here $a = 1$, $b = -2$, and $c = -5$, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\,(1)\,(-5)}}{2\,(1)} = \frac{2 \pm \sqrt{24}}{2} = 1 \pm \sqrt{6}.$$

34. We use the quadratic formula to solve the equation $2x^2 + 8x + 7 = 0$. Here $a = 2$, $b = 8$, and $c = 7$, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4\,(2)\,(7)}}{4} = \frac{-8 \pm 2\sqrt{2}}{4} = -2 \pm \tfrac{1}{2}\sqrt{2}.$$

35. $$\frac{(t+6)\,(60) - (60t + 180)}{(t+6)^2} = \frac{60t + 360 - 60t - 180}{(t+6)^2} = \frac{180}{(t+6)^2}.$$

36. $$\frac{6x}{2\left(3x^2+2\right)} + \frac{1}{4\,(x+2)} = \frac{(6x)\,(2)\,(x+2) + \left(3x^2+2\right)}{4\left(3x^2+2\right)(x+2)} = \frac{12x^2 + 24x + 3x^2 + 2}{4\left(3x^2+2\right)(x+2)} = \frac{15x^2 + 24x + 2}{4\left(3x^2+2\right)(x+2)}.$$

37. $$\frac{2}{3}\left(\frac{4x}{2x^2-1}\right) + 3\left(\frac{3}{3x-1}\right) = \frac{8x}{3\left(2x^2-1\right)} + \frac{9}{3x-1} = \frac{8x\,(3x-1) + 27\left(2x^2-1\right)}{3\left(2x^2-1\right)(3x-1)} = \frac{78x^2 - 8x - 27}{3\left(2x^2-1\right)(3x-1)}.$$

38. $$\frac{-2x}{\sqrt{x+1}} + 4\sqrt{x+1} = \frac{-2x + 4\,(x+1)}{\sqrt{x+1}} = \frac{2\,(x+2)}{\sqrt{x+1}}.$$

39. $$\frac{\sqrt{x}-1}{x-1} = \frac{\sqrt{x}-1}{x-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{\left(\sqrt{x}\right)^2 - 1}{(x-1)\left(\sqrt{x}+1\right)} = \frac{x-1}{(x-1)\left(\sqrt{x}+1\right)} = \frac{1}{\sqrt{x}+1}.$$

40. $\dfrac{\sqrt{x}-1}{2\sqrt{x}} = \dfrac{\sqrt{x}-1}{2\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{x-\sqrt{x}}{2x}$.

41. The distance is $d = \sqrt{[1-(-2)]^2 + [-7-(-3)]^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$.

42. The distance is $d = \sqrt{(2-6)^2 + (6-9)^2} = \sqrt{16+9} = \sqrt{25} = 5$.

43. The distance is $d = \sqrt{\left(-\frac{1}{2} - \frac{1}{2}\right)^2 + \left(2\sqrt{3} - \sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$.

44. An equation is $x = -2$.

45. An equation is $y = 4$.

46. The slope of L is $m = \dfrac{\frac{7}{2} - 4}{3-(-2)} = -\dfrac{1}{10}$, and an equation of L is $y - 4 = -\frac{1}{10}[x-(-2)] = -\frac{1}{10}x - \frac{1}{5}$, or $y = -\frac{1}{10}x + \frac{19}{5}$. The general form of this equation is $x + 10y - 38 = 0$.

47. The line passes through the points $(-2, 4)$ and $(3, 0)$, so its slope is $m = \dfrac{4-0}{-2-3} = -\dfrac{4}{5}$. An equation is $y - 0 = -\frac{4}{5}(x-3)$, or $y = -\frac{4}{5}x + \frac{12}{5}$.

48. Writing the given equation in the form $y = \frac{5}{2}x - 3$, we see that the slope of the given line is $\frac{5}{2}$. Thus, an equation is $y - 4 = \frac{5}{2}(x+2)$, or $y = \frac{5}{2}x + 9$. The general form of this equation is $5x - 2y + 18 = 0$.

49. Writing the given equation in the form $y = -\frac{4}{3}x + 2$, we see that the slope of the given line is $-\frac{4}{3}$. Therefore, the slope of the required line is $\frac{3}{4}$ and an equation of the line is $y - 4 = \frac{3}{4}(x+2)$ or $y = \frac{3}{4}x + \frac{11}{2}$.

50. Rewriting the given equation in slope-intercept form, we have $4y = -3x + 8$ or $y = -\frac{3}{4}x + 2$. We conclude that the slope of the required line is $-\frac{3}{4}$. Using the point-slope form of the equation of a line with the point $(2, 3)$ and slope $-\frac{3}{4}$, we obtain $y - 3 = -\frac{3}{4}(x-2)$, and so $y = -\frac{3}{4}x + \frac{6}{4} + 3 = -\frac{3}{4}x + \frac{9}{2}$. The general form of this equation is $3x + 4y - 18 = 0$.

51. The slope of the line joining the points $(-3, 4)$ and $(2, 1)$ is $m = \dfrac{1-4}{2-(-3)} = -\dfrac{3}{5}$. Using the point-slope form of the equation of a line with the point $(-1, 3)$ and slope $-\frac{3}{5}$, we have $y - 3 = -\frac{3}{5}[x-(-1)]$. Therefore, $y = -\frac{3}{5}(x+1) + 3 = -\frac{3}{5}x + \frac{12}{5}$.

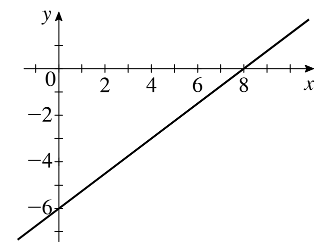
52. The slope of the line passing through $(-2, -4)$ and $(1, 5)$ is $m = \dfrac{5-(-4)}{1-(-2)} = \dfrac{9}{3} = 3$, so the required line is $y - (-2) = 3[x-(-3)]$. Simplifying, this is equivalent to $y + 2 = 3x + 9$, or $y = 3x + 7$.

53. Rewriting the given equation in the slope-intercept form $y = \frac{2}{3}x - 8$, we see that the slope of the line with this equation is $\frac{2}{3}$. The slope of a line perpendicular to this line is thus $-\frac{3}{2}$. Using the point-slope form of the equation of a line with the point $(-2, -4)$ and slope $-\frac{3}{2}$, we have $y - (-4) = -\frac{3}{2}[x-(-2)]$ or $y = -\frac{3}{2}x - 7$. The general form of this equation is $3x + 2y + 14 = 0$.

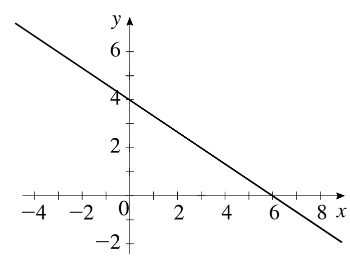
54. Substituting $x = -1$ and $y = -\frac{5}{4}$ into the left-hand side of the equation gives $6(-1) - 8\left(-\frac{5}{4}\right) - 16 = -12$. The equation is not satisfied, and so we conclude that the point $\left(-1, -\frac{5}{4}\right)$ does not lie on the line $6x - 8y - 16 = 0$.

55. Substituting $x = 2$ and $y = -4$ into the equation, we obtain $2(2) + k(-4) = -8$, so $-4k = -12$ and $k = 3$.

56. Setting $x = 0$ gives $y = -6$ as the y-intercept. Setting $y = 0$ gives $x = 8$ as the x-intercept. The graph of $3x - 4y = 24$ is shown.



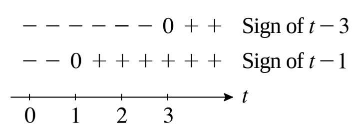
57. Using the point-slope form of an equation of a line, we have $y - 2 = -\frac{2}{3}(x - 3)$ or $y = -\frac{2}{3}x + 4$. If $y = 0$, then $x = 6$, and if $x = 0$, then $y = 4$. A sketch of the line is shown.



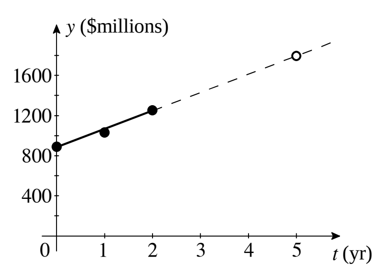
58. Simplifying $2(1.5C + 80) \le 2(2.5C - 20)$, we obtain $1.5C + 80 \le 2.5C - 20$, so $C \ge 100$ and the minimum cost is \$100.

59. $3(2R - 320) \le 3R + 240$ gives $6R - 960 \le 3R + 240$, $3R \le 1200$ and finally $R \le 400$. We conclude that the maximum revenue is \$400.

60. We solve the inequality $-16t^2 + 64t + 80 \ge 128$, obtaining $-16t^2 + 64t - 48 \ge 0$, $t^2 - 4t + 3 \le 0$, and $(t - 3)(t - 1) \le 0$. From the sign diagram, we see that the required solution is $[1, 3]$. Thus, the stone is 128 ft or higher off the ground between 1 and 3 seconds after it was thrown.



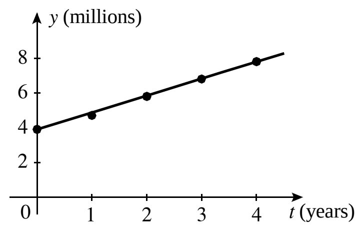
61. a, b.



c. The slope of L is $\dfrac{1251 - 887}{2 - 0} = 182$, so an equation of L is $y - 887 = 182(t - 0)$ or $y = 182t + 887$.

d. The amount consumers are projected to spend on Cyber Monday, 2014 ($t = 5$) is $182(5) + 887$, or \$1.797 billion.

62. a, b.



c. $P_1(0, 3.9)$ and $P_2(4, 7.8)$, so $m = \dfrac{7.8 - 3.9}{4 - 0} = \dfrac{3.9}{4} = 0.975$. Thus, $y - 3.9 = 0.975(t - 0)$, or $y = 0.975t + 3.9$.

d. If $t = 3$, then $y = 0.975(3) + 3.9 = 6.825$. Thus, the number of systems installed in 2005 (when $t = 3$) is 6,825,000, which is close to the projected value of 6.8 million.

CHAPTER 1 Before Moving On... page 50

1. a. $\left|\pi - 2\sqrt{3}\right| - \left|\sqrt{3} - \sqrt{2}\right| = -\left(\pi - 2\sqrt{3}\right) - \left(\sqrt{3} - \sqrt{2}\right) = \sqrt{3} + \sqrt{2} - \pi$.

b. $\left[\left(-\frac{1}{3}\right)^{-3}\right]^{1/3} = \left(-\frac{1}{3}\right)^{(-3)\left(\frac{1}{3}\right)} = \left(-\frac{1}{3}\right)^{-1} = -3$.

2. a. $\sqrt[3]{64x^6} \cdot \sqrt{9y^2x^6} = \left(4x^2\right)\left(3yx^3\right) = 12x^5y$.

b. $\left(\dfrac{a^{-3}}{b^{-4}}\right)^2 \left(\dfrac{b}{a}\right)^{-3} = \dfrac{a^{-6}}{b^{-8}} \cdot \dfrac{b^{-3}}{a^{-3}} = \dfrac{b^8}{a^6} \cdot \dfrac{a^3}{b^3} = \dfrac{b^5}{a^3}$.

3. a. $\dfrac{2x}{3\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{2x\sqrt{y}}{3y}$.

b. $\dfrac{x}{\sqrt{x} - 4} \cdot \dfrac{\sqrt{x} + 4}{\sqrt{x} + 4} = \dfrac{x\left(\sqrt{x} + 4\right)}{x - 16}$.

4. a. $\dfrac{\left(x^2 + 1\right)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}(2x)}{\left(x^2 + 1\right)^2} = \dfrac{\frac{1}{2}x^{-1/2}\left[\left(x^2 + 1\right) - 4x^2\right]}{\left(x^2 + 1\right)^2} = \dfrac{1 - 3x^2}{2x^{1/2}\left(x^2 + 1\right)^2}$.

b. $-\dfrac{3x}{\sqrt{x + 2}} + 3\sqrt{x + 2} = \dfrac{-3x + 3(x + 2)}{\sqrt{x + 2}} = \dfrac{6}{\sqrt{x + 2}} = \dfrac{6\sqrt{x + 2}}{x + 2}$.

5. $\dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} \cdot \dfrac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \dfrac{x - y}{\left(\sqrt{x} - \sqrt{y}\right)^2}$.

6. a. $12x^3 - 10x^2 - 12x = 2x\left(6x^2 - 5x - 6\right) = 2x(2x - 3)(3x + 2)$.

b. $2bx - 2by + 3cx - 3cy = 2b(x - y) + 3c(x - y) = (2b + 3c)(x - y)$

7. a. $12x^2 - 9x - 3 = 0$, so $3\left(4x^2 - 3x - 1\right) = 0$ and $3(4x + 1)(x - 1) = 0$. Thus, $x = -\frac{1}{4}$ or $x = 1$.

b. $3x^2 - 5x + 1 = 0$. Using the quadratic formula with $a = 3$, $b = -5$, and $c = 1$, we have
$x = \dfrac{-(-5) \pm \sqrt{25 - 12}}{2(3)} = \dfrac{5 \pm \sqrt{13}}{6}$.

8. $d = \sqrt{[6 - (-2)]^2 + (8 - 4)^2} = \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$.

9. $m = \dfrac{5 - (-2)}{4 - (-1)} = \dfrac{7}{5}$, so $y - (-2) = \frac{7}{5}[x - (-1)]$, $y + 2 = \frac{7}{5}x + \frac{7}{5}$, or $y = \frac{7}{5}x - \frac{3}{5}$.

10. $m = -\frac{1}{3}$ and $b = \frac{4}{3}$, so an equation is $y = -\frac{1}{3}x + \frac{4}{3}$.

CHAPTER 1 Explore & Discuss

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1. Let $P_1 = (2, 6)$ and $P_2 = (-4, 3)$. Then we have $x_1 = 2$, $y_1 = 6$, $x_2 = -4$, and $y_2 = 3$. Using Formula (1), we have $d = \sqrt{(-4-2)^2 + (3-6)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$, as obtained in Example 1.

2. Let $P_1\,(x_1, y_1)$ and $P_2\,(x_2, y_2)$ be any two points in the plane. Then the result follows from the equality $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$.

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1. **a.** All points on and inside the circle with center (h, k) and radius r.

b. All points inside the circle with center (h, k) and radius r.

c. All points on and outside the circle with center (h, k) and radius r.

d. All points outside the circle with center (h, k) and radius r.

2. **a.** $y^2 = 4 - x^2$, and so $y = \pm\sqrt{4-x^2}$.

b. **(i)** The upper semicircle with center at the origin and radius 2.

(ii) The lower semicircle with center at the origin and radius 2.

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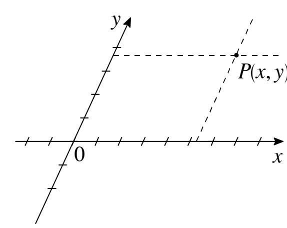
1. Let $P\,(x, y)$ be any point in the plane. Draw a line through P parallel to the y-axis and a line through P parallel to the x-axis (see the figure). The x-coordinate of P is the number corresponding to the point on the x-axis at which the line through P crosses the x-axis. Similarly, y is the number that corresponds to the point on the y-axis at which the line parallel to the x-axis crosses the y-axis. To show the converse, reverse the process.



2. You can use the Pythagorean Theorem in the Cartesian coordinate system. This greatly simplifies the computations.

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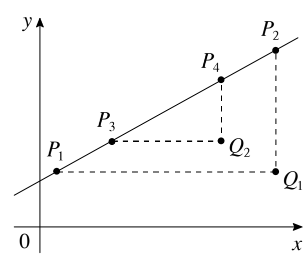
1. Refer to the accompanying figure. Observe that triangles $\Delta P_1 Q_1 P_2$ and $\Delta P_3 Q_2 P_4$ are similar. From this we conclude that $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_4 - y_3}{x_4 - x_3}$. Because P_3 and P_4 are arbitrary, the conclusion follows.



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1. We obtain a family of parallel lines each having slope m.

2. We obtain a family of straight lines all of which pass through the point $(0, b)$.

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1. In Example 11, we are told that the object is expected to appreciate in value at a given rate for the next five years, and the equation obtained in that example is based on this fact. Thus, the equation may not be used to predict the value of the object very much beyond five years from the date of purchase.

CHAPTER 1 Exploring with Technology

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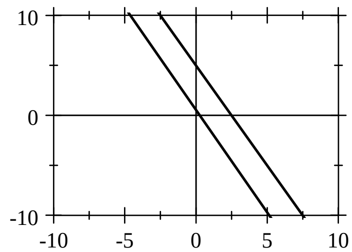
1.

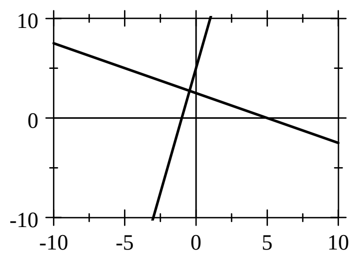


The straight lines L_1 and L_2 are shown in the figure.

a. L_1 and L_2 seem to be parallel.

b. Writing each equation in the slope-intercept form gives $y = -2x + 5$ and $y = -\frac{41}{20}x + \frac{11}{20}$, from which we see that the slopes of L_1 and L_2 are -2 and $-\frac{41}{20} = -2.05$, respectively. This shows that L_1 and L_2 are not parallel.

2.



The straight lines L_1 and L_2 are shown in the figure.

a. L_1 and L_2 seem to be perpendicular.

b. The slopes of L_1 and L_2 are $m_1 = -\frac{1}{2}$ and $m_2 = 5$, respectively. Because $m_1 = -\dfrac{1}{2} \neq -\dfrac{1}{5} = -\dfrac{1}{m_2}$, we see that L_1 and L_2 are not perpendicular.

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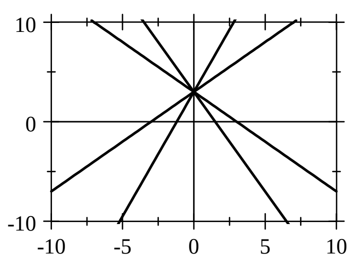
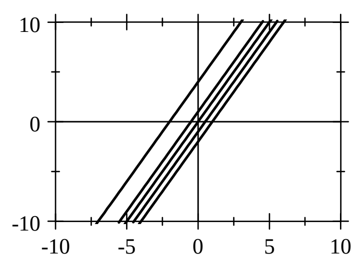
1.



The straight lines with the given equations are shown in the figure. Changing the value of m in the equation $y = mx + b$ changes the slope of the line and thus rotates it.

2.



The straight lines of interest are shown in the figure. Changing the value of b in the equation $y = mx + b$ changes the y-intercept of the line and thus translates it (upward if $b > 0$ and downward if $b < 0$).

3. Changing both m and b in the equation $y = mx + b$ both rotates and translates the line.